UNIVERSITY OF TWENTE.

Formal Methods & Tools.

Efficient Modelling and Generation of Probabilistic Automata as well as Markov Automata

Mark Timmer

June 29, 2012

Joint work with Joost-Pieter Katoen, Jaco van de Pol, and Mariëlle Stoelinga

The context: probabilistic model checking

Probabilistic model checking:

- Verifying quantitative properties,
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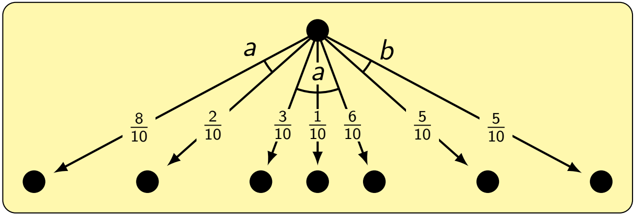

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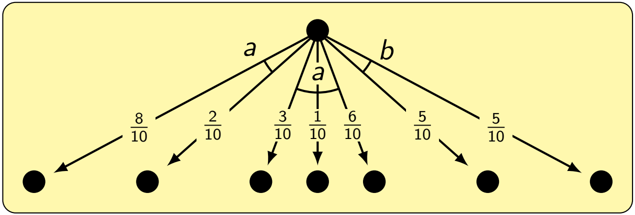

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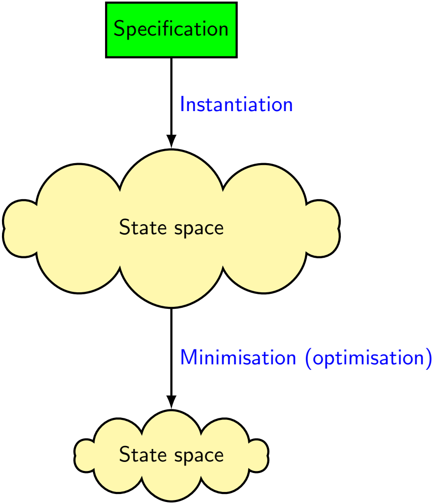
Limitations of previous approaches:

- Susceptible to the state space explosion problem
- Restricted treatment of data

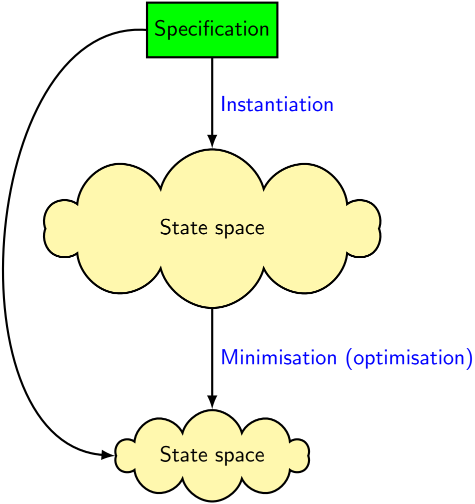
Combating the state space explosion



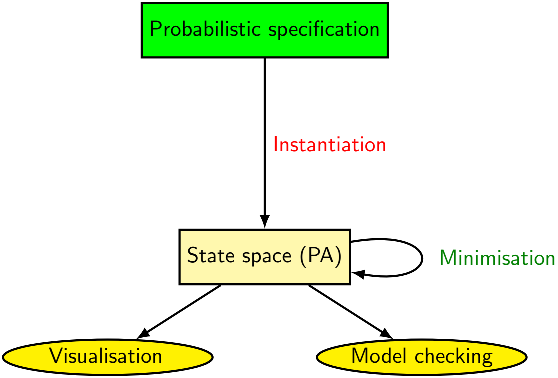
Combating the state space explosion

Optimised instantiation

- Dead variable reduction
- Confluence reduction



Overview of our approach



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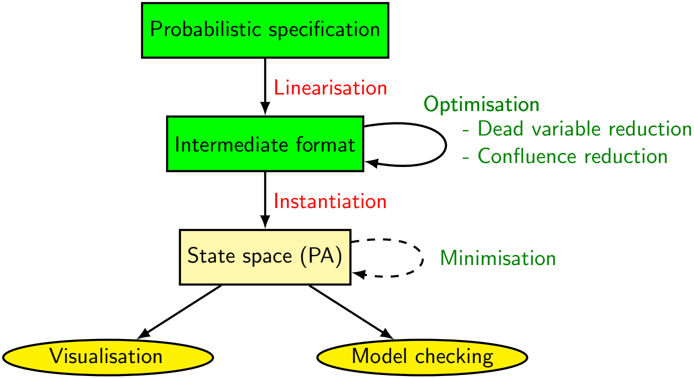

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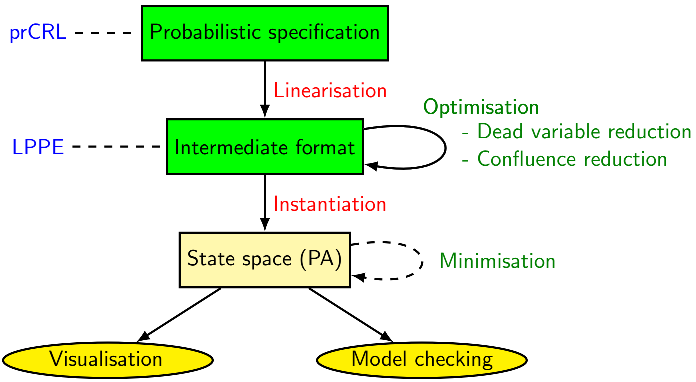
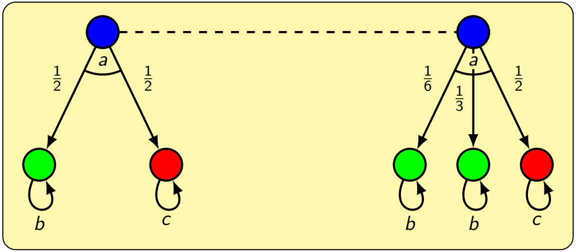


Strong bisimulation for Probabilistic Automata

Mimic behaviour with equal probabilities:



Contents

A process algebra with data and probability: prCRL

Specification language prCRL:

- Based on μCRL (so data), with additional probabilistic choice
- Semantics defined in terms of probabilistic automata
- Minimal set of operators to facilitate formal manipulation
- Syntactic sugar easily definable

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The grammar of prCRL process terms

Process terms in prCRL are obtained by the following grammar:

$$p ::= Y(\boldsymbol{t}) \mid c \Rightarrow p \mid p + p \mid \sum_{x:D} p \mid a(\boldsymbol{t}) \sum_{x:D}\!\!\bullet\, f : p$$

Process equations and processes

A process equation is something of the form $X(\boldsymbol{g} : \boldsymbol{G}) = p$.

An example specification

Sending an arbitrary natural number

$$X(\text{active} : \text{Bool}) =$$

$$\text{not}(\text{active}) \Rightarrow \text{ping} \cdot \sum_{b:\text{Bool}} X(b)$$

$$+ \text{active} \Rightarrow \tau \sum_{n:\mathbb{N}^{>0}}\!\!\bullet\; \tfrac{1}{2^n} : \left(\text{send}(n) \cdot X(\text{false})\right)$$

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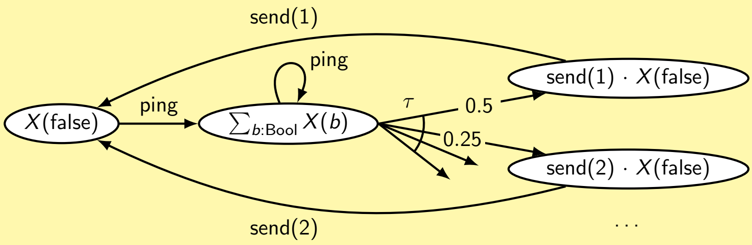

Composability using extended prCRL

For composability we introduce extended prCRL. It extends prCRL by parallel composition, encapsulation, hiding and renaming.

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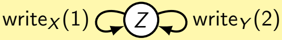

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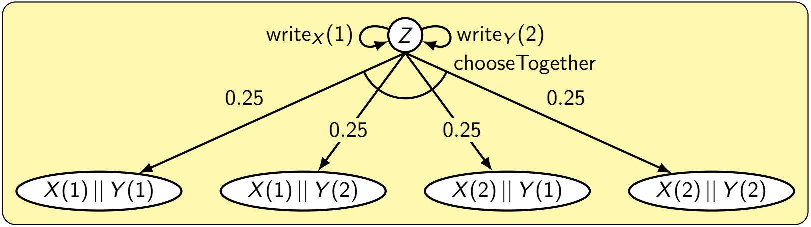

A linear format for prCRL: the LPPE

LPPEs are a subset of prCRL specifications:

$$X(\boldsymbol{g} : \boldsymbol{G}) = \sum_{\boldsymbol{d_1} : \boldsymbol{D_1}} c_1 \Rightarrow a_1(b_1) \sum\!\!\!\bullet_{\boldsymbol{e_1} : \boldsymbol{E_1}} f_1 : X(\boldsymbol{n_1})$$
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Advantages of using LPPEs instead of prCRL specifications:

- Easy state space generation
- Straight-forward parallel composition
- **Symbolic optimisations enabled at the language level**

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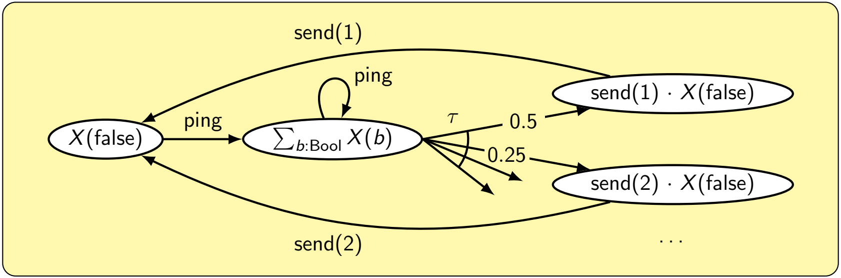
Theorem

Every specification (without unguarded recursion) can be linearised to an LPPE, preserving strong probabilistic bisimulation.

Linear Probabilistic Process Equations – an example



Specification in prCRL

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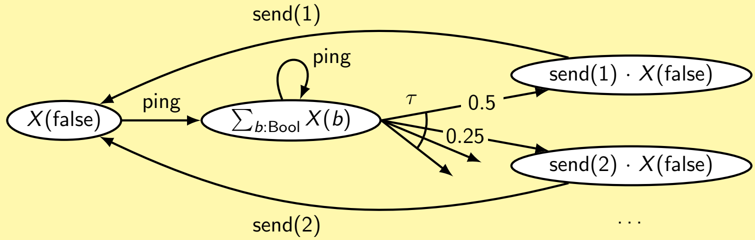

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Specification in LPPE

$$X(pc : \{1..3\}, n : \mathbb{N}^{\geq 0}) =$$
$$+\ pc = 1 \Rightarrow \text{ping} \cdot X(2, 1)$$
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Linearisation: a simple example without data

Consider the following prCRL specification:

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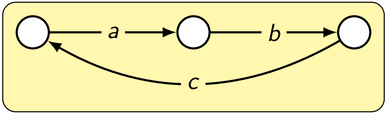

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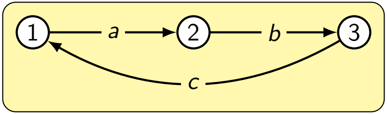

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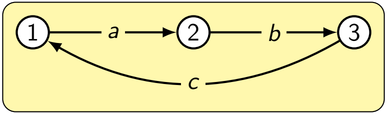

The corresponding LPPE (initialised with pc = 1):

$$\begin{aligned} Y(pc\colon \{1,2,3\}) = & \\ & pc = 1 \Rightarrow a \cdot Y(2) \\ + & pc = 2 \Rightarrow b \cdot Y(3) \\ + & pc = 3 \Rightarrow c \cdot Y(1) \end{aligned}$$

Linearisation: a more complicated example with data

Consider the following prCRL specification:

$$X = \sum_{d:D} \text{get}(d) \cdot (\tau \cdot \text{loss} \cdot X + \tau \cdot \text{send}(d) \cdot X)$$

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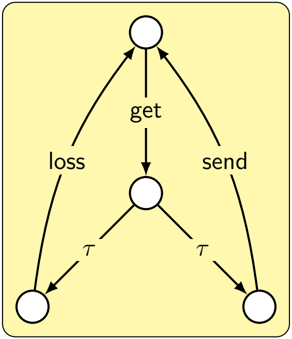

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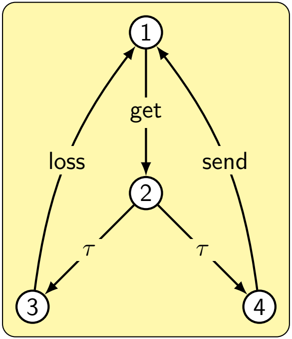

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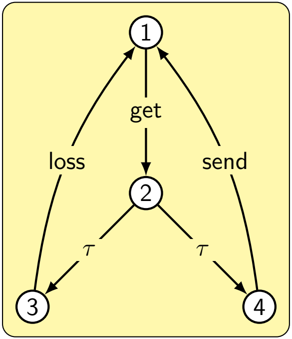

LPPE:

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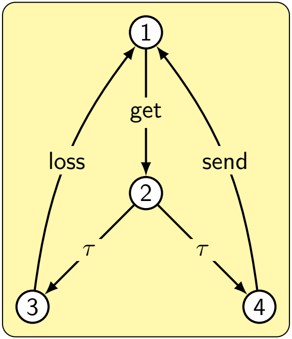

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Initial process: $Y(1, d_1)$.

Linearisation: a more algorithmic approach

Consider the following prCRL specification:

$$X(d : D) = \sum_{e:D} a(d+e) \sum_{f:D}\!\!\!\!\!\bullet\;\; \frac{1}{|D|} : \left(c(e) \cdot c(f) \cdot X(5) + c(e + f) \cdot X(5) \right)$$

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$$\begin{aligned} X(\mathrm{pc}:\{1,2,3\}, d:D, e:D, f:D) = {} & \\ & \mathrm{pc}=1 \Rightarrow \sum_{e:D} a(d+e) \sum_{f:D}\!\!\bullet\ \tfrac{1}{|D|}\cdot X(2,d,e,f) \\ & + \mathrm{pc}=2 \Rightarrow c(e)\cdot X(3,d,e,f) \\ & + \mathrm{pc}=2 \Rightarrow c(e+f)\cdot X(1,5,e,f) \\ & + \mathrm{pc}=3 \Rightarrow c(f)\cdot X(1,5,e,f) \end{aligned}$$

Linearisation

In general, we always linearise in two steps:

1. Transform the specification to intermediate regular form (IRF) (every process is a summation of single-action terms)
2. Merge all processes into one big process by introducing a program counter

In the first step, global parameters are introduced to remember the values of bound variables.

Reductions techniques for LPPEs

1. LPPE simplification techniques
    - Constant elimination
    - Summation elimination
    - Expression simplification

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$X(id : Id) = print(id) \cdot X(id)$

init $X(Mark)$

$\rightarrow$

$X = print(Mark) \cdot X$

init X

Reductions techniques for LPPEs

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   - Constant elimination
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$X = \sum_{d:\{1,2,3\}} d = 2 \Rightarrow send(d) \cdot X$

init X

$\rightarrow$

$X = send(2) \cdot X$

init X

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$X = (3 = 1 + 2 \vee x > 5) \Rightarrow beep \cdot Y$ $\rightarrow$ $X = beep \cdot Y$

Reductions techniques for LPPEs

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- Deduce the control flow of an LPPE
- Examine relevance (liveness) of variables
- Reset dead variables

Reductions techniques for LPPEs

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- Detect confluent internal transitions
- Give these transitions priority

Intermediate summary

What you heard so far

- We developed the process algebra prCRL, incorporating both data and probability;
- We defined a normal form for prCRL, the LPPE; starting point for symbolic optimisations and easy state space generation;
- We provided a linearisation algorithm to transform prCRL specifications to LPPEs, proved it correct and implemented it;
- We developed several reduction techniques for LPPEs that preserve strong/branching probabilistic bisimulation.

Contents

The overall goal: efficient and expressive modelling

Specifying systems with

- Nondeterminism ⟵ LTSs
- Probability ⟵ DTMCs
- Stochastic timing ⟵ CTMCs

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Nondeterminism and Probability → Probabilistic Automata (PAs)

The overall goal: efficient and expressive modelling

Specifying systems with

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Interactive Markov Chains (IMCs) (Nondeterminism, Stochastic timing)

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Markov Automata (MAs)

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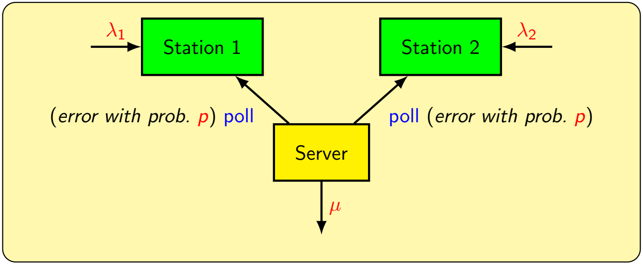

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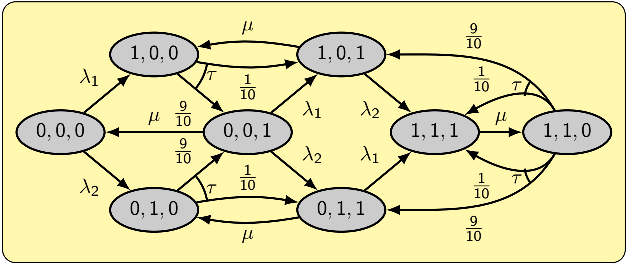

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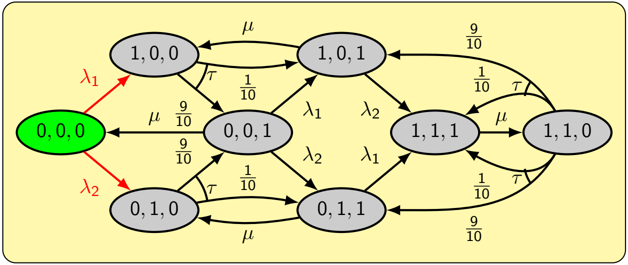

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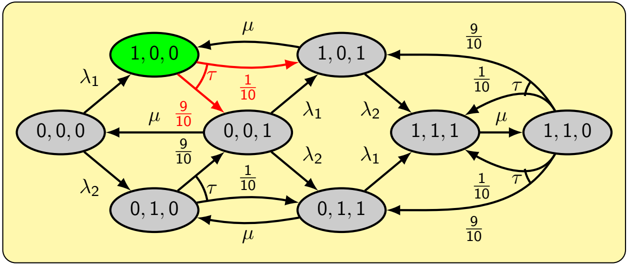

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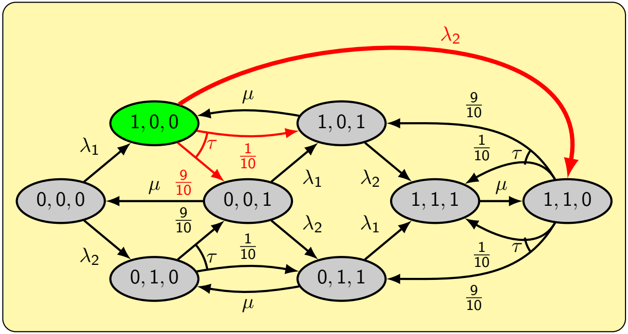

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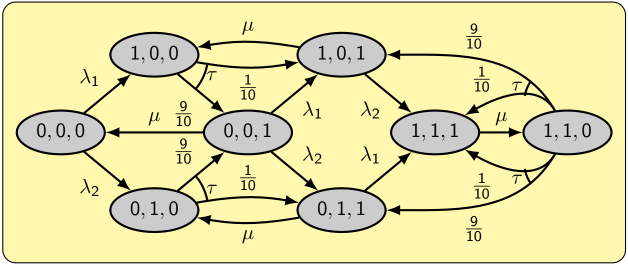

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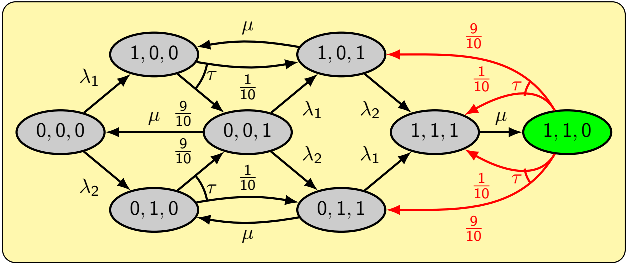

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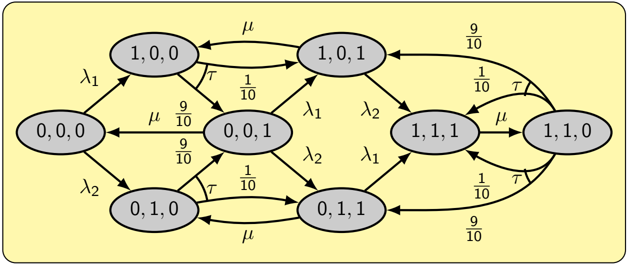

Observed limitations:

- No easy process-algebraic modelling language with data
- Susceptible to the state space explosion problem

Approach: extending and reusing

PA	$\rightarrow$	MA

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prCRL	$\rightarrow$	MAPA	(Markov Automata Process Algebra)

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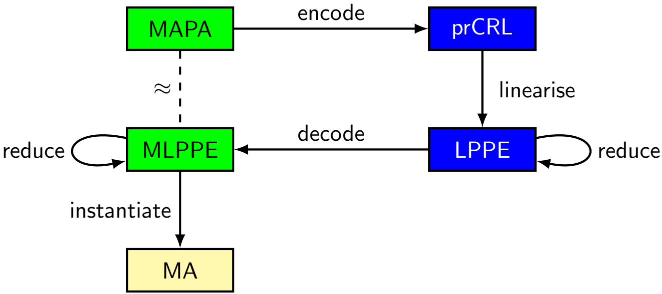
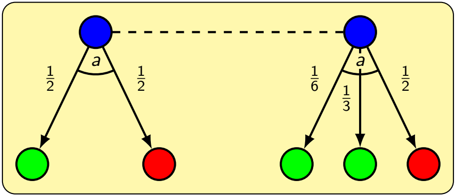


Strong bisimulation for Markov automata

Mimic interactive behaviour:



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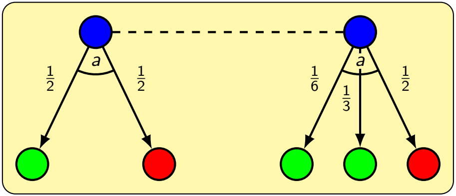

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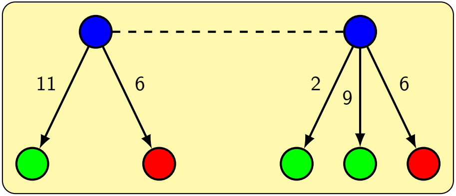

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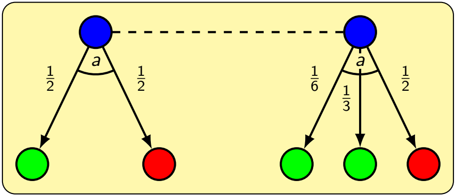

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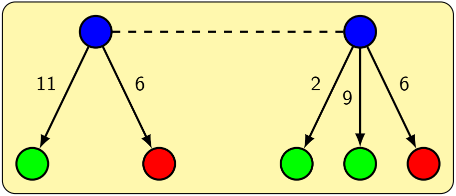

(If a state enables a τ-transition, all rates are ignored.)

A process algebra with data for MAs: MAPA

Specification language MAPA:

- Based on prCRL: data and probabilistic choice
- Additional feature: Markovian rates
- Semantics defined in terms of Markov automata
- Minimal set of operators to facilitate formal manipulation
- Syntactic sugar easily definable

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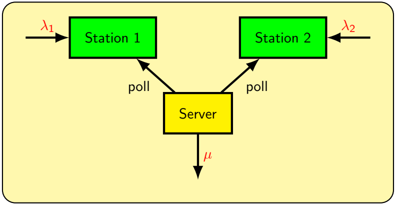
The grammar of MAPA

Process terms in MAPA are obtained by the following grammar:

$$p ::= Y(\boldsymbol{t}) \mid c \Rightarrow p \mid p + p \mid \sum_{\boldsymbol{x}:\boldsymbol{D}} p \mid a(\boldsymbol{t}) \sum\!\!\!\!\bullet_{\boldsymbol{x}:\boldsymbol{D}} f : p \mid (\lambda) \cdot p$$

An example specification



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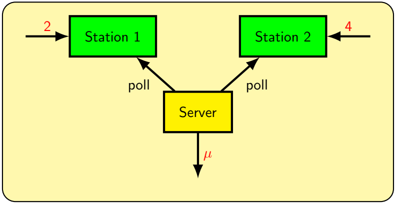

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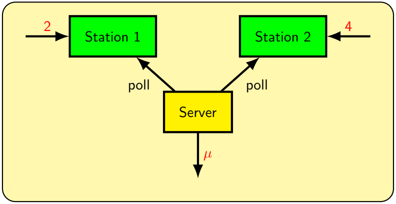

- There are 10 types of jobs
- The type of job that arrives is chosen nondeterministically
- Service time depends on job type (hence, we need queues)

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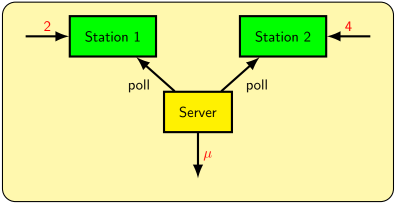

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The specification of the stations:

type *Jobs* $= \{1, \ldots, 10\}$

Station$(i : \{1,2\}, q : \text{Queue})$
$= \mathbf{notFull}(q) \quad \Rightarrow (2i) \cdot \sum_{j:Jobs} arrive(j).Station(i, \mathbf{enqueue}(q, j))$

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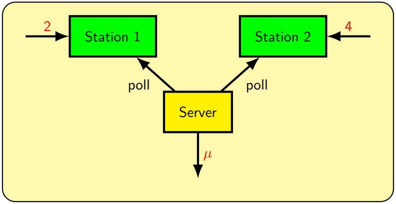

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$+ \mathbf{notEmpty}(q) \Rightarrow \mathit{deliver}(i, \mathbf{head}(q)) \sum_{k \in \{1,9\}} \frac{k}{10} : k = 1 \Rightarrow \mathit{Station}(i, q)$
$+ k = 9 \Rightarrow \mathit{Station}(i, \mathbf{tail}(q))$

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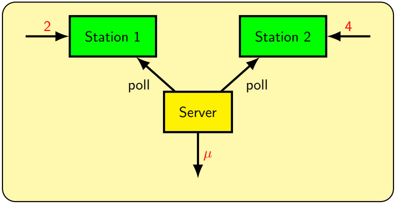

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Derivation-based operational semantics

$$\textsc{MarkovPrefix}\ \frac{-}{(\lambda)\cdot p \xrightarrow{\lambda} p} \qquad \textsc{SumLeft}\ \frac{p \xrightarrow{a} p'}{p + q \xrightarrow{a} p'}$$

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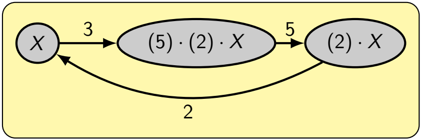

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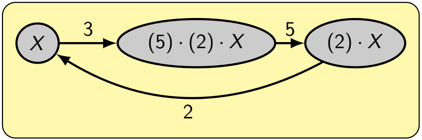
$$X = (3)\cdot(5)\cdot X + c\cdot X$$

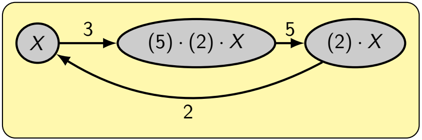


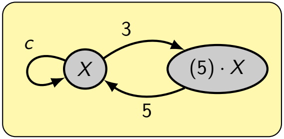
Derivation-based operational semantics

$$\textsc{MarkovPrefix}\ \frac{-}{(\lambda) \cdot p \xrightarrow{\lambda} p} \qquad \textsc{SumLeft}\ \frac{p \xrightarrow{a} p'}{p + q \xrightarrow{a} p'}$$

$$X = (3) \cdot (5) \cdot (2) \cdot X$$

$$X = (3) \cdot (5) \cdot X + c \cdot X$$





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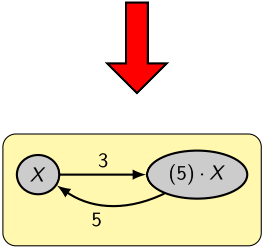

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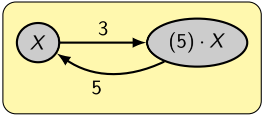

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Derivation-based operational semantics

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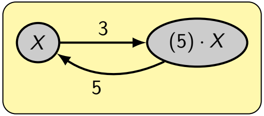

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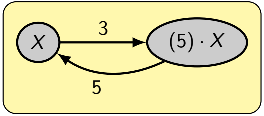

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Hence, the total rate from X to $(5)\cdot X$ is $3+3=6$.

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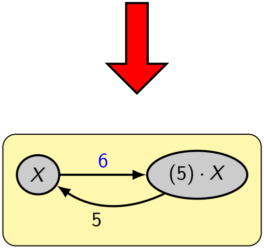

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MLPPEs

We defined a special format for MAPA, the MLPPE:

$$X(\boldsymbol{g} : \boldsymbol{G}) = \sum_{i \in I} \sum_{\boldsymbol{d_i} : \boldsymbol{D_i}} c_i \Rightarrow a_i(\boldsymbol{b_i}) \sum_{\bullet\, \boldsymbol{e_i} : \boldsymbol{E_i}} f_i : X(\boldsymbol{n_i})$$

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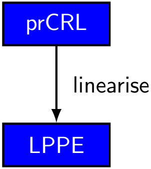
Advantages of using MLPPEs instead of MAPA specifications:

- Easy state space generation
- Straight-forward parallel composition
- **Symbolic optimisations enabled at the language level**

Encoding into prCRL



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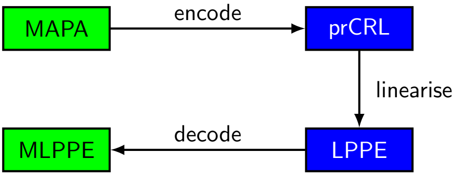

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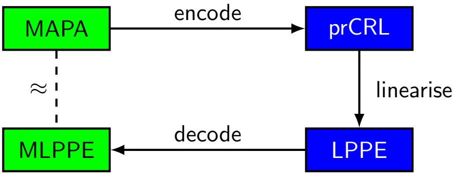

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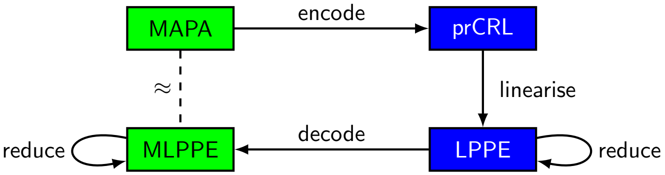

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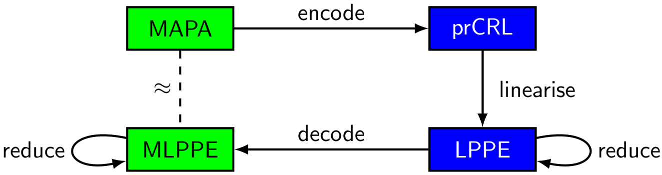

Basic idea: encode a rate λ as action rate(λ).

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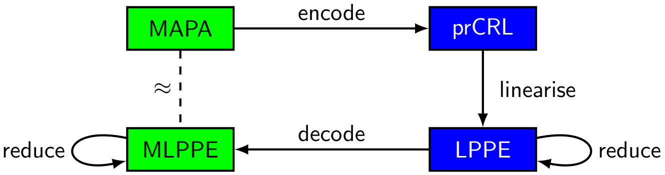

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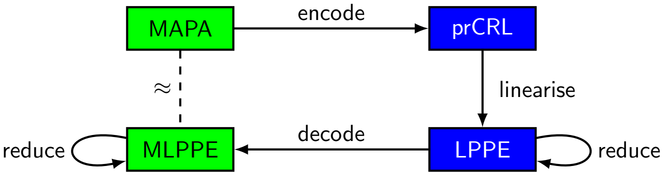

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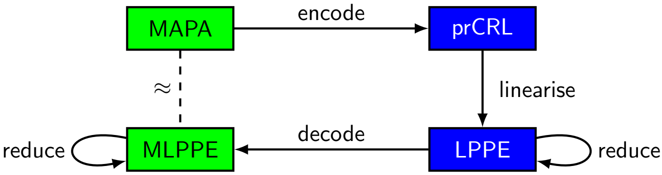

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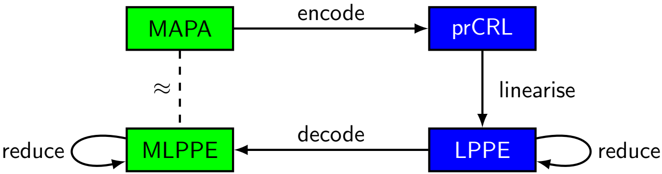

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$$\approx_{\text{PA}}$$

$$\text{rate}(\lambda) \cdot p$$

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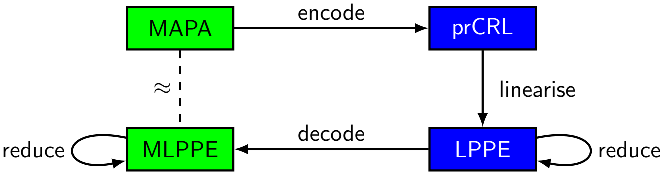

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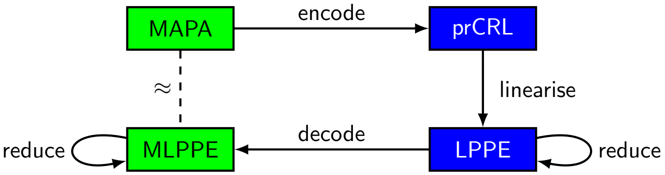

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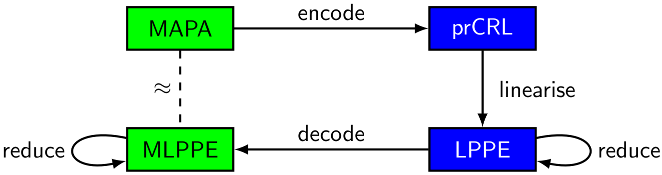

Possible solution: encode a rate λ as action rate$_i(\lambda)$.

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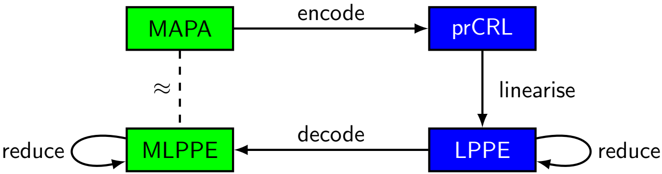

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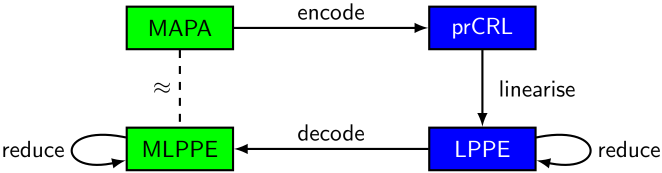

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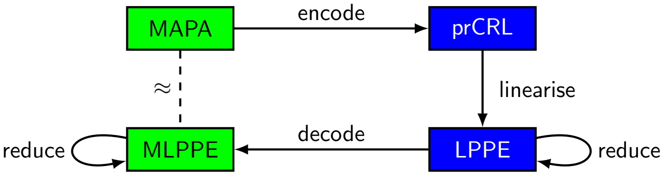

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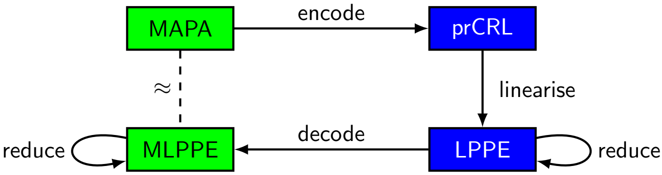

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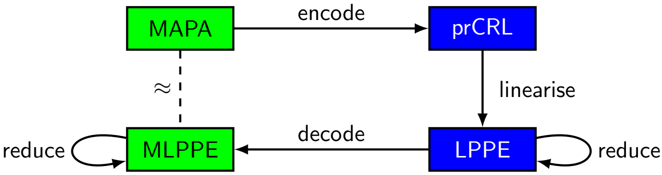

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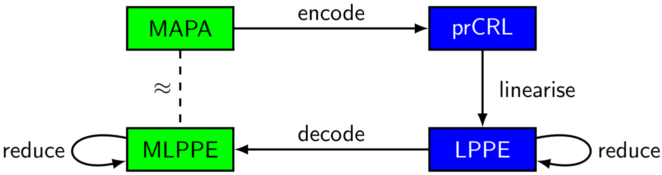

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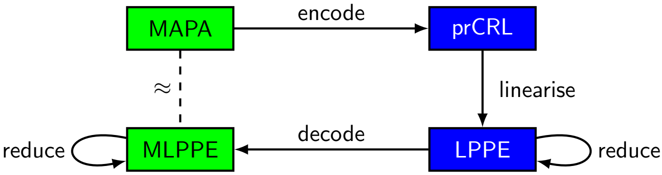

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Stronger equivalence on prCRL specifications needed!

Derivation-preserving bisimulation

Two prCRL terms are derivation-preserving bisimulation if

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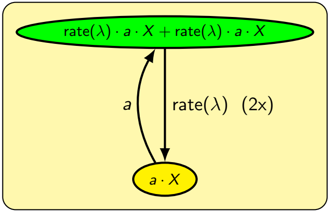
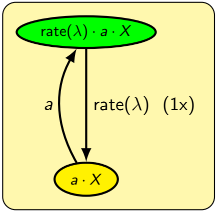


$\not\approx_{\text{dp}}$



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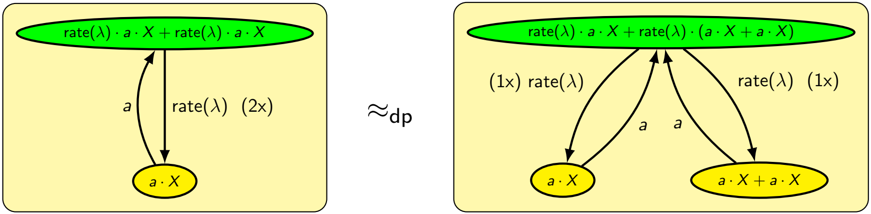

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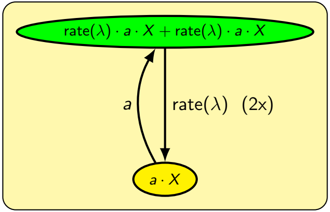

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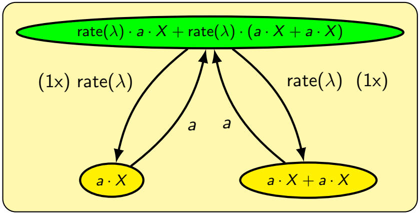

Proposition

Derivation-preserving bisimulation is a congruence for prCRL.

Derivation-preserving bisimulation: important results

Theorem

Given a derivation-preserving prCRL transformation f,

$$\text{decode}(f(\text{encode}(M))) \approx M$$

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Corollary

The linearisation procedure of prCRL can be reused for MAPA.

Generalising existing reduction techniques

Existing reduction techniques that preserve derivations:

- Constant elimination
- Expression simplification
- Dead variable reduction

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$$\text{deadVarRed} = \text{decode} \circ \text{deadVarRedOld} \circ \text{encode}$$

Novel reduction techniques

New reduction techniques for MAPA:

- Maximal progress reduction
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$X = \tau \cdot X + (5) \cdot X$ $\rightarrow$ $X = \tau \cdot X$

Novel reduction techniques

New reduction techniques for MAPA:

- Maximal progress reduction
- Summation elimination

$$X = \sum_{d:\{1,2,3\}} d = 2 \Rightarrow send(d) \cdot X$$

$$Y = \sum_{d:\{1,2,3\}} (5) \cdot Y$$

$\rightarrow$

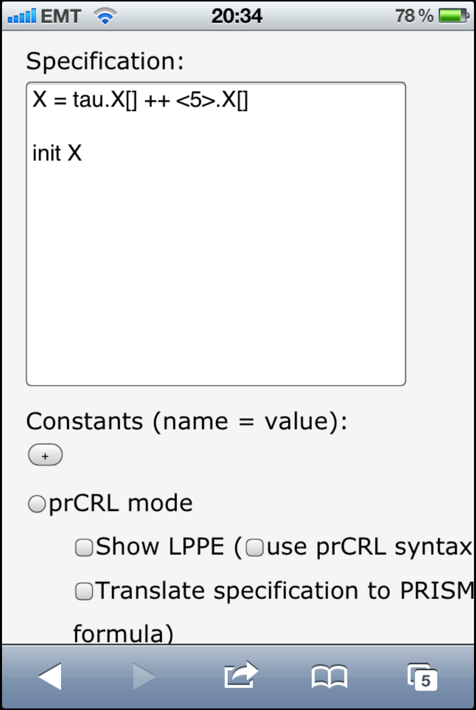
$$X = send(2) \cdot X$$

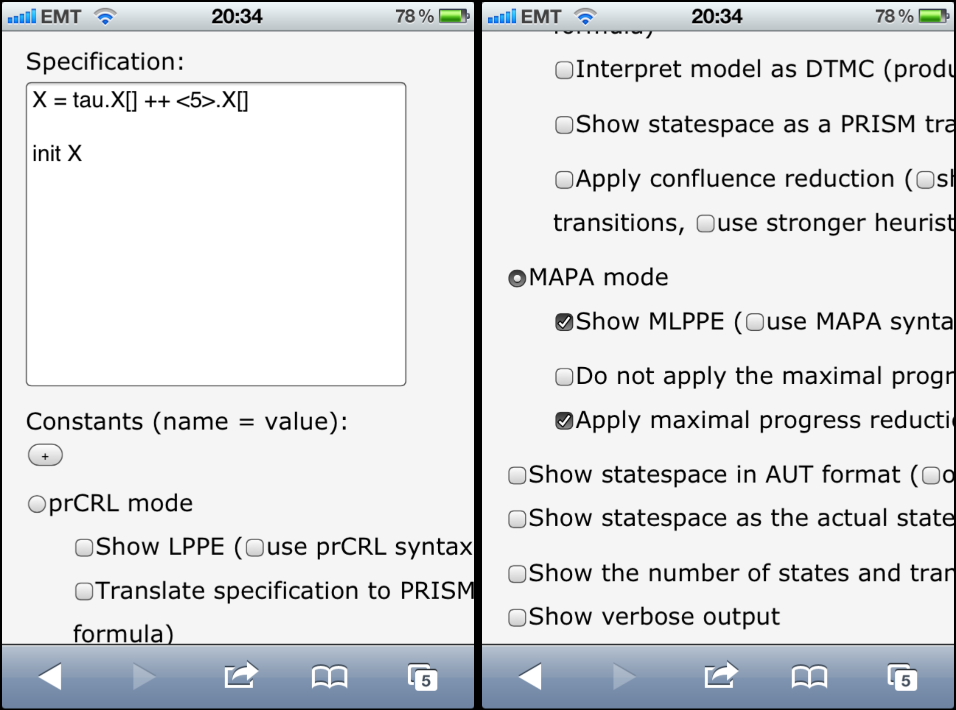
$$Y = (15) \cdot Y$$

Implementation and Case Study

Implementation in SCOOP:

- Programmed in Haskell
- Stand-alone and web-based interface
- Linearisation, optimisation, state space generation


EMT
20:34
78 %
Specification:
X = tau.X[] ++ <5>.X[]
init X
Constants (name = value):
+
prCRL mode
Show LPPE (use prCRL syntax
Translate specification to PRISM
formula)
5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EMT
20:34
78 %
Interpret model as DTMC (produ
Show statespace as a PRISM tra
Apply confluence reduction (sh
transitions, use stronger heurist
MAPA mode
Show MLPPE (use MAPA synta
Do not apply the maximal progr
Apply maximal progress reducti
Show statespace in AUT format (o
Show statespace as the actual state
Show the number of states and tran
Show verbose output
5
5

EMT 20:35 78 %

Apply unused variable reduction

☐ Apply dead variable reduction

☐ Apply transition merging

☐ Suppress all basic (M)LPPE reductio

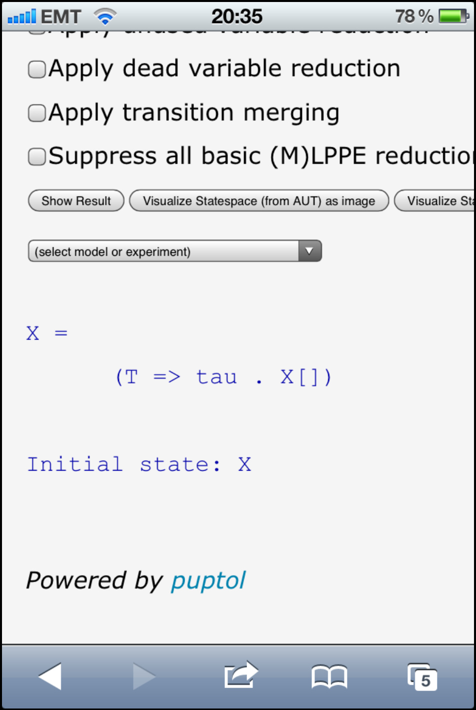
Show Result | Visualize Statespace (from AUT) as image | Visualize St

(select model or experiment)

```
X =
       (T => tau . X[])


Initial state: X
```

Powered by puptol


EMT
20:35
78 %
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Suppress all basic (M)LPPE reductio
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Visualize Sta
(select model or experiment)
X =
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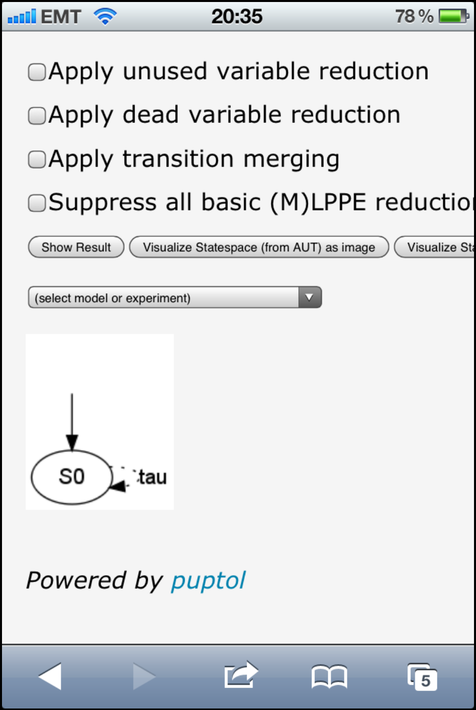
Initial state: X
Powered by puptol
5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	Original				Reduced				
Spec.	States	Trans.	MLPPE	Time	States	Trans.	MLPPE	Time	Red.
queue-3-5	316,058	581,892	15 / 335	87.4	218,714	484,548	8 / 224	20.7	76%
queue-3-6	1,005,699	1,874,138	15 / 335	323.3	670,294	1,538,733	8 / 224	64.7	80%
queue-3-6'	1,005,699	1,874,138	15 / 335	319.5	74	108	5 / 170	0.0	100%
queue-5-2	27,659	47,130	15 / 335	4.3	23,690	43,161	8 / 224	1.9	56%
queue-5-3	1,191,738	2,116,304	15 / 335	235.8	926,746	1,851,312	8 / 224	84.2	64%
queue-5-3'	1,191,738	2,116,304	15 / 335	233.2	170	256	5 / 170	0.0	100%
queue-25-1	3,330	5,256	15 / 335	0.5	3,330	5,256	8 / 224	0.4	20%
queue-100-1	50,805	81,006	15 / 335	8.9	50,805	81,006	8 / 224	6.6	26%
mutex-3-2	17,352	40,200	27 / 3,540	12.3	10,560	25,392	12 / 2,190	4.6	63%
mutex-3-4	129,112	320,136	27 / 3,540	95.8	70,744	169,128	12 / 2,190	30.3	68%
mutex-3-6	425,528	1,137,048	27 / 3,540	330.8	224,000	534,624	12 / 2,190	99.0	70%
mutex-4-1	27,701	80,516	36 / 5,872	33.0	20,025	62,876	16 / 3,632	13.5	59%
mutex-4-2	360,768	1,035,584	36 / 5,872	435.9	218,624	671,328	16 / 3,632	145.5	67%
mutex-4-3	1,711,141	5,015,692	36 / 5,872	2,108.0	958,921	2,923,300	16 / 3,632	644.3	69%
mutex-5-1	294,882	1,051,775	45 / 8,780	549.7	218,717	841,750	20 / 5,430	216.6	61%

Table: State space generation using SCOOP.

Implementation and Case Study

Implementation in SCOOP:

- Programmed in Haskell
- Stand-alone and web-based interface
- Linearisation, optimisation, state space generation

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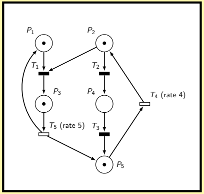
GSPN analysis

GSPN (PNML)



`reach P1 = 1 & P5 = 2`

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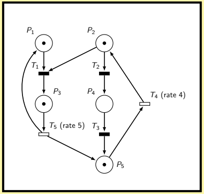


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GEMMA

MAPA

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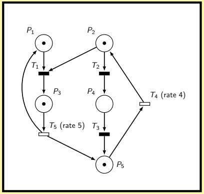

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SCOOP

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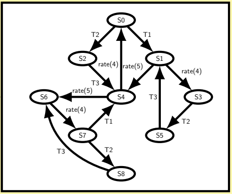

`#GOALS S4`

GSPN analysis

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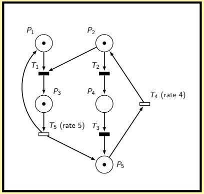

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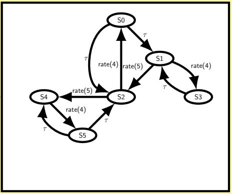
SCOOP (optimised)

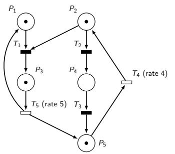
MA



`#GOALS S2`

GSPN analysis

GSPN (PNML)



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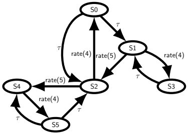
SCOOP (optimised)

MA



`#GOALS S2`

IMCA

Results

Min. unbounded reach.:	1.0
Max. unbounded reach.:	1.0
Min. expected time:	0.0
Max. expected time:	0.2
Min. LRA:	0.0
Max. LRA:	0.4

`#GOALS S2`

Conclusions and Future Work

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Future Work:

- Generalise confluence reduction to MAs and MAPA

Questions

Questions?

Have a look at `fmt.cs.utwente.nl/~timmer/scoop`