

UNIVERSITY OF TWENTE.

Formal Methods & Tools.

**Why Confluence is More Powerful
than Ample Sets in Probabilistic
and Non-Probabilistic Branching Time**

Mark Timmer

May 23, 2012

The context – probabilistic model checking

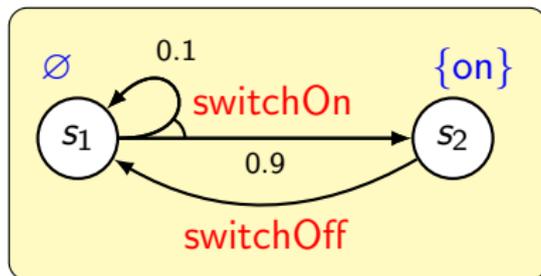
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- Verifying **quantitative properties**,
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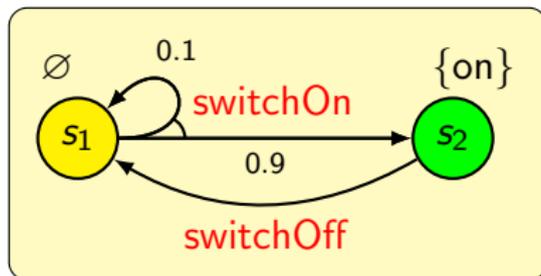


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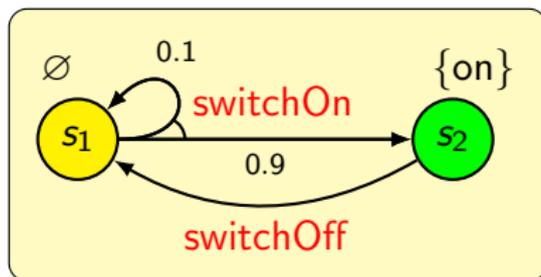


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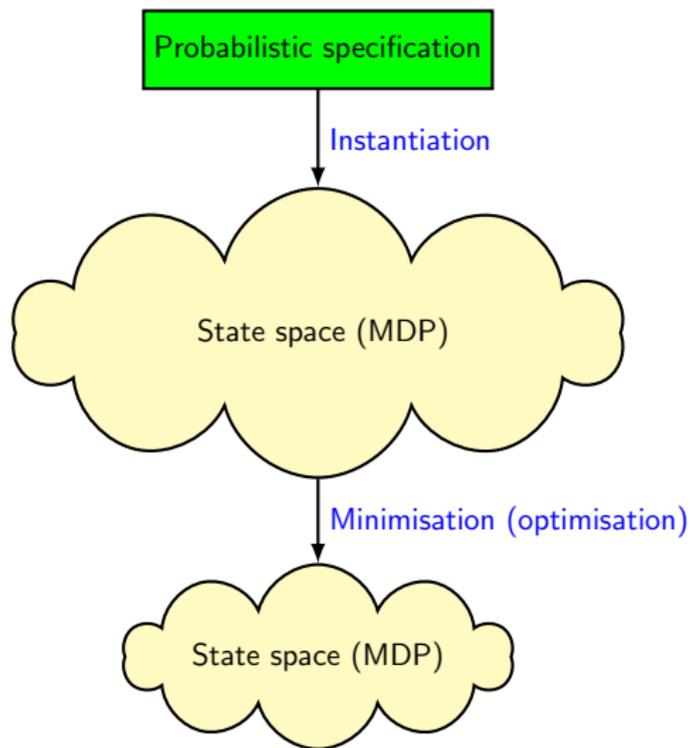


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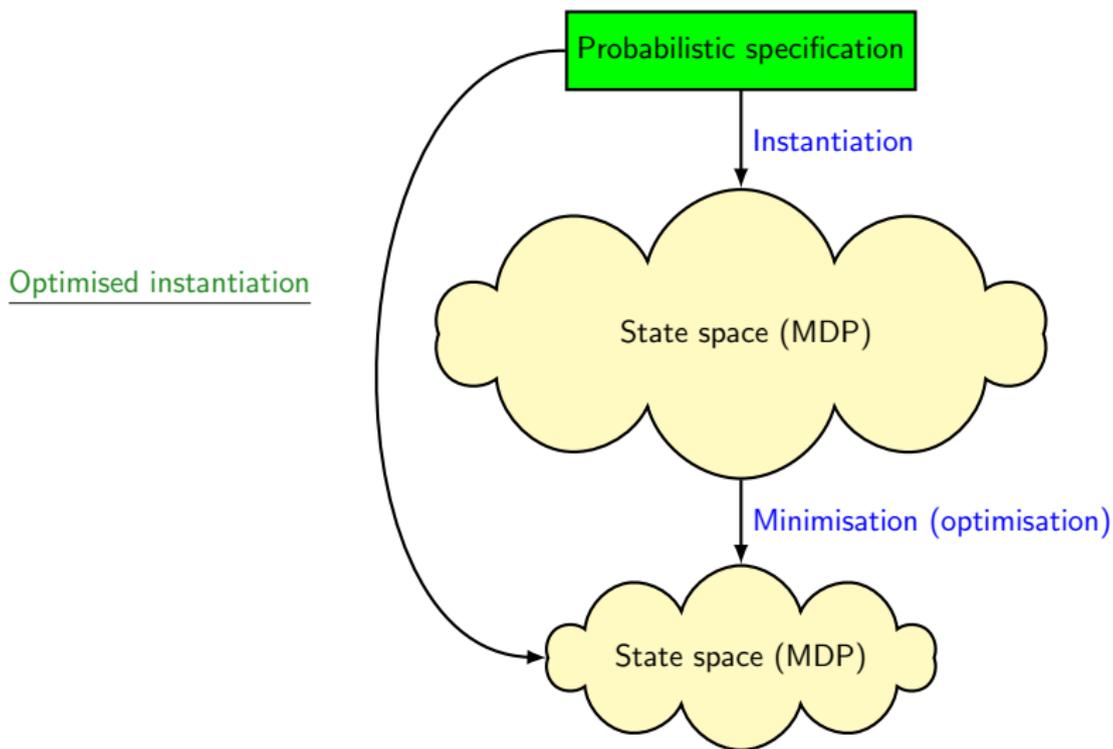
Main limitation (as for non-probabilistic model checking):

- Susceptible to the **state space explosion** problem

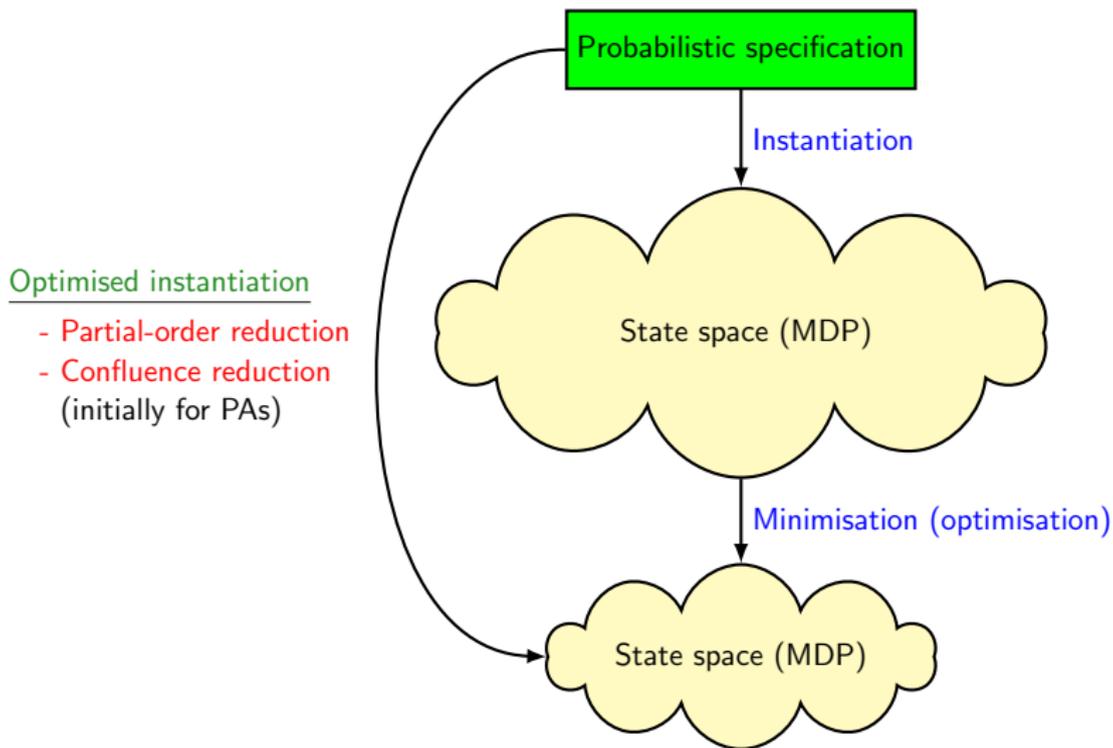
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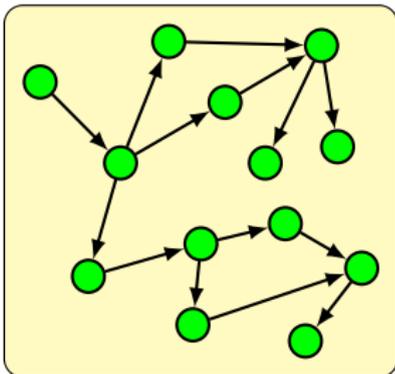
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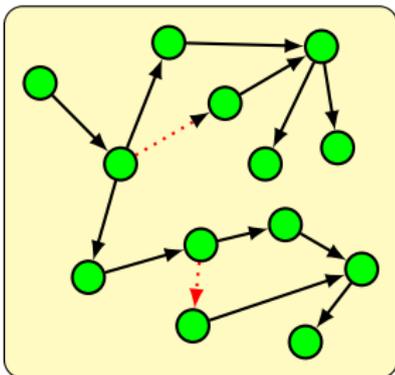
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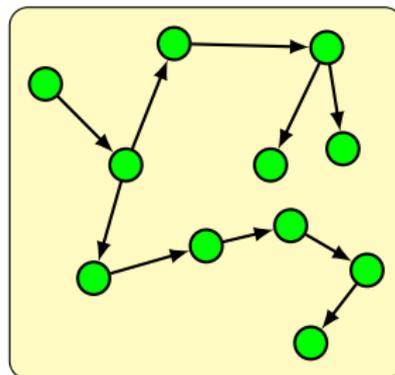
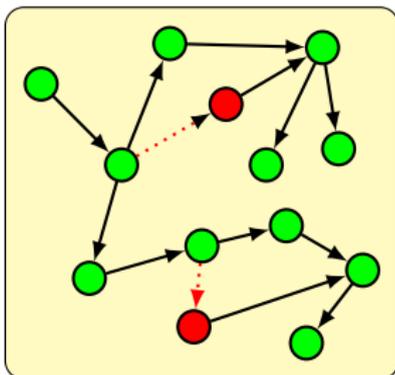
Reductions – an overview



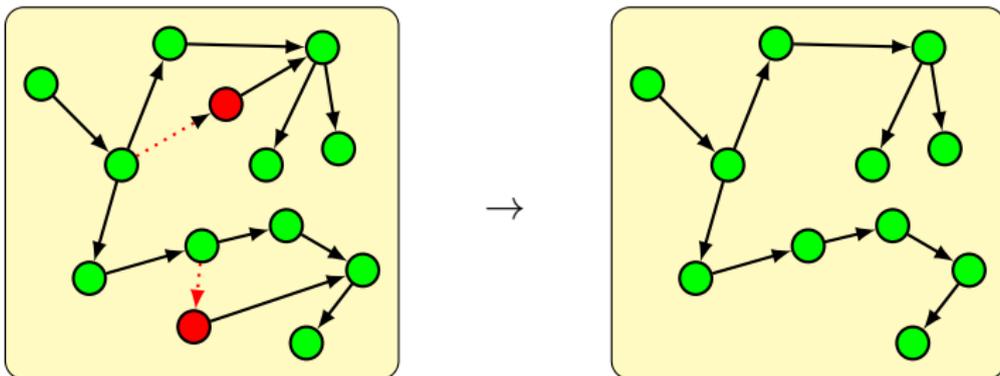
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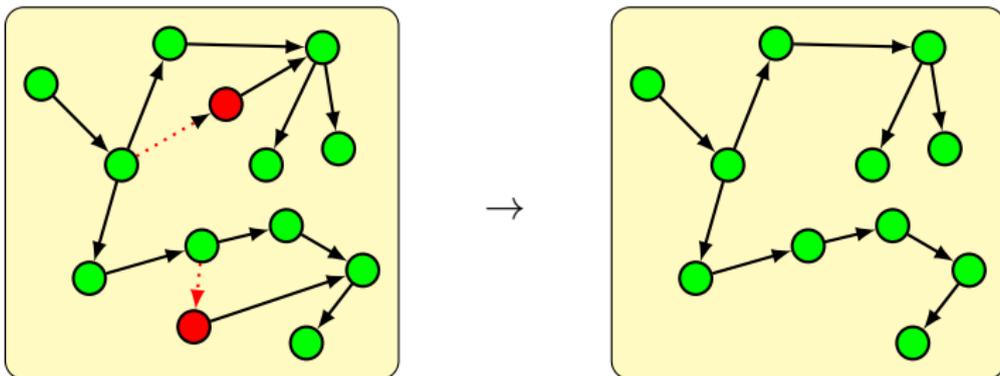
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Reduction function:

$$R: S \rightarrow 2^{\Sigma}$$

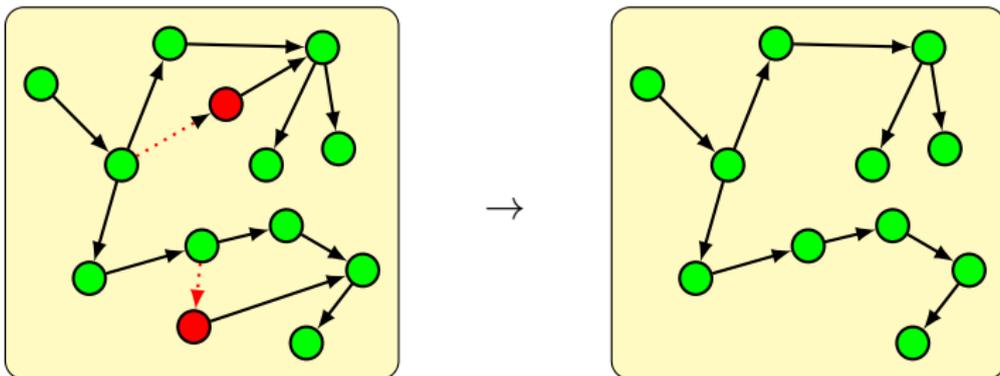
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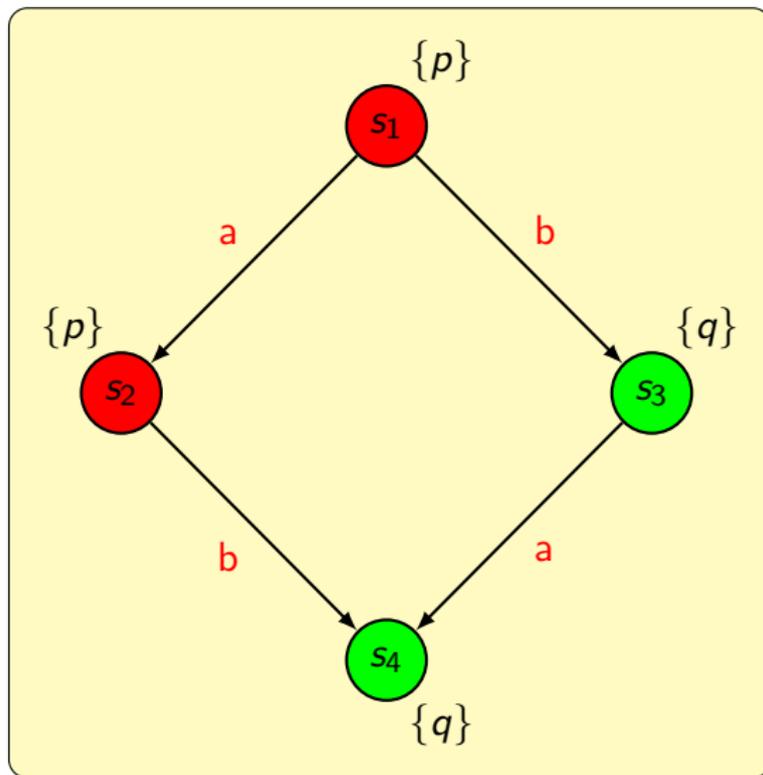


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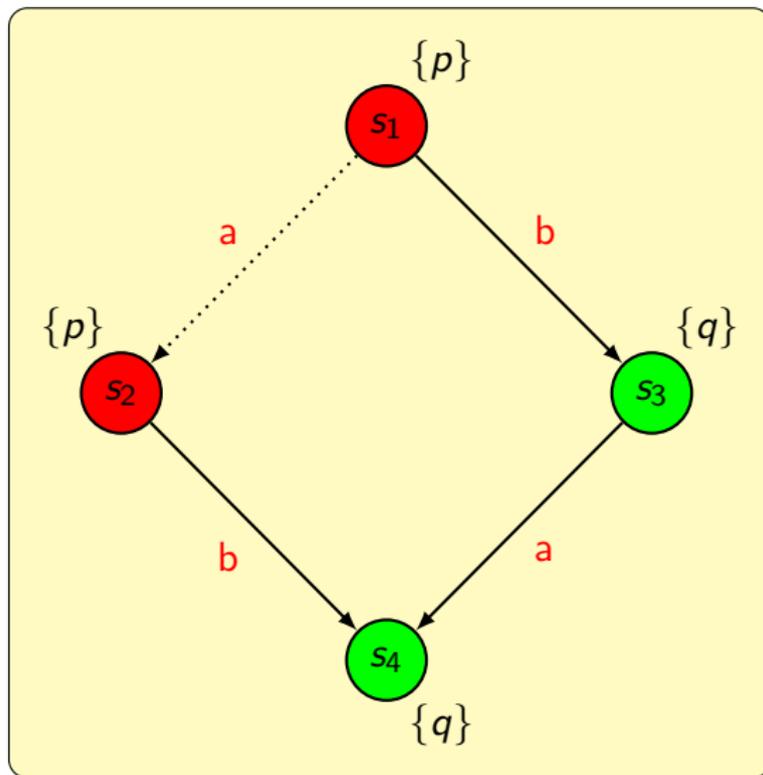
$$R: S \rightarrow 2^{\Sigma} \quad (R(s) \subseteq \text{enabled}(s))$$

If $R(s) \neq \text{enabled}(s)$, then $R(s)$ consists of **remaining transitions**.

Basic concepts



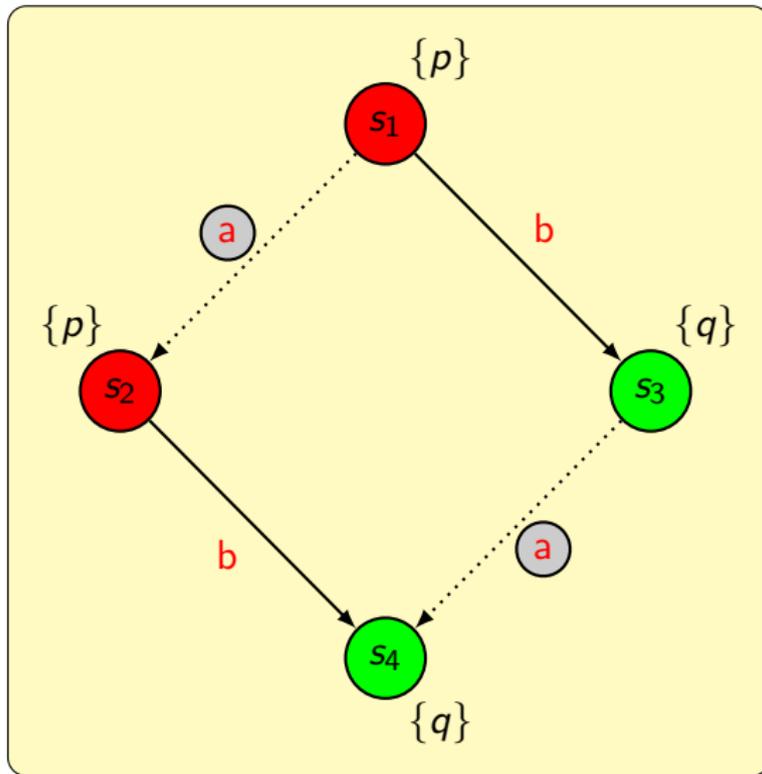
Basic concepts



Stuttering transition:

- No **observable change**

Basic concepts



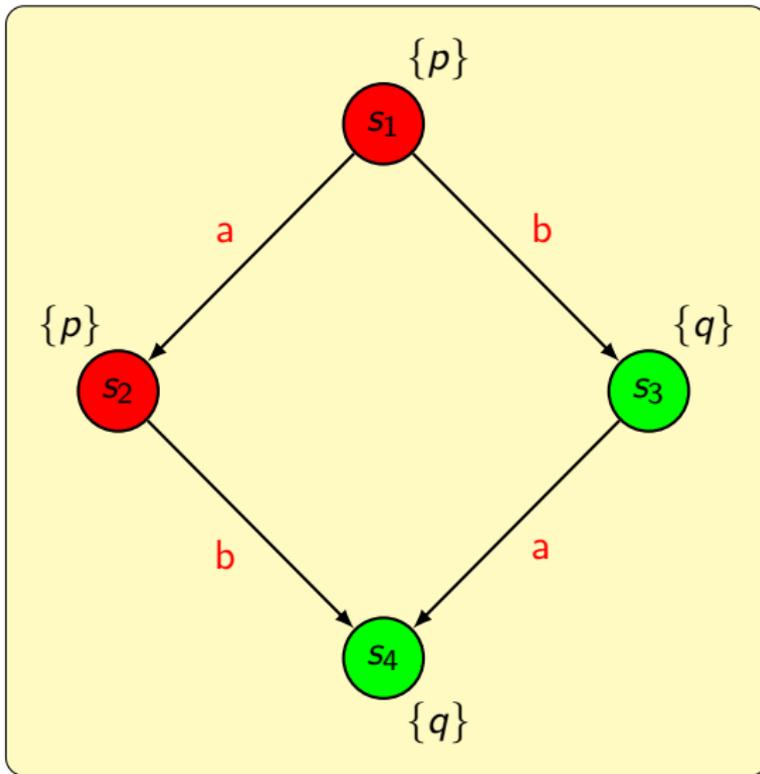
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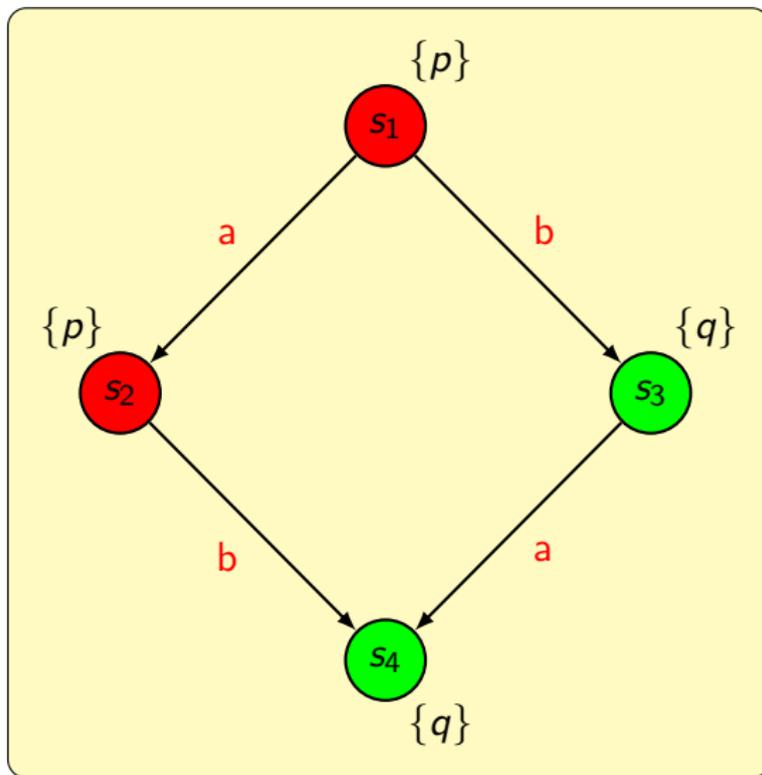
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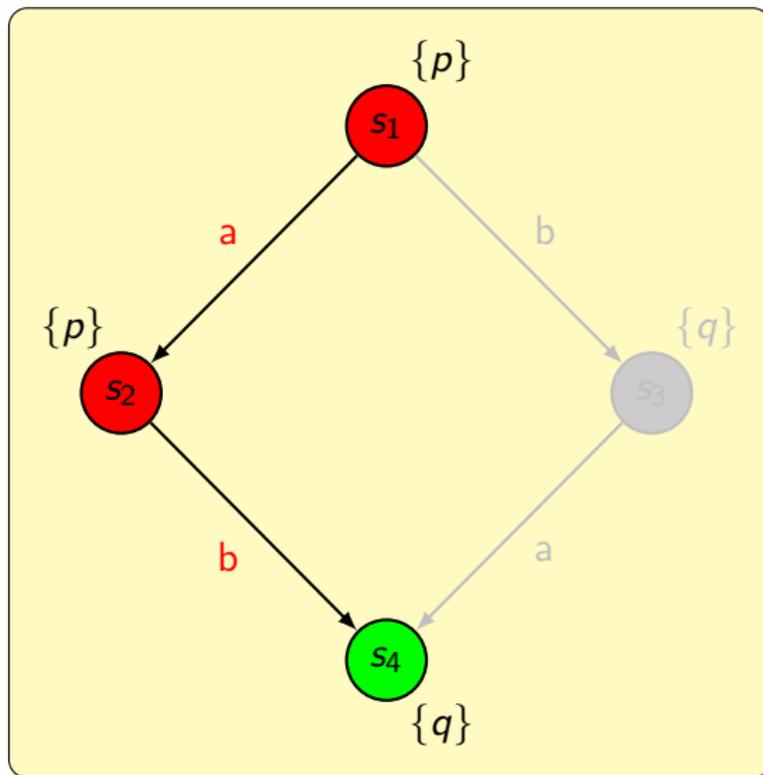
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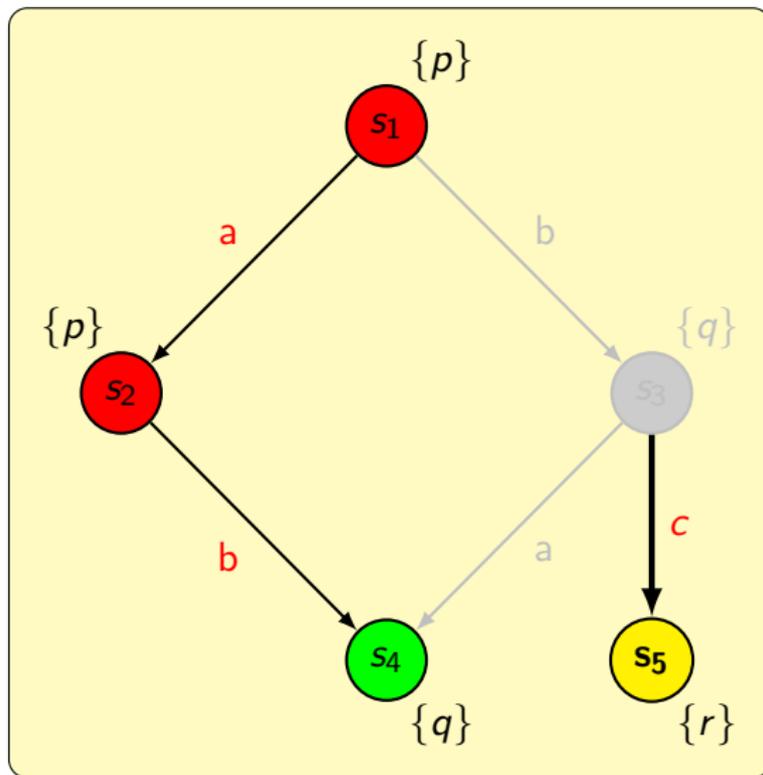
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Correctness criteria

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- Preservation of $LTL_{\setminus X}$ (linear time)
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Partial-order reduction: ample sets

Partial-order reduction [Baier, D'Argenio, Größer, 2006]

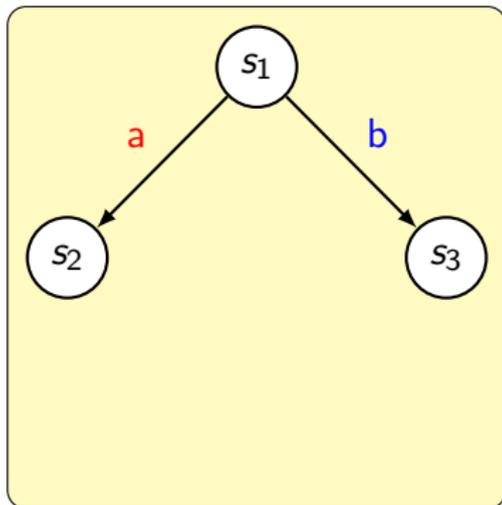
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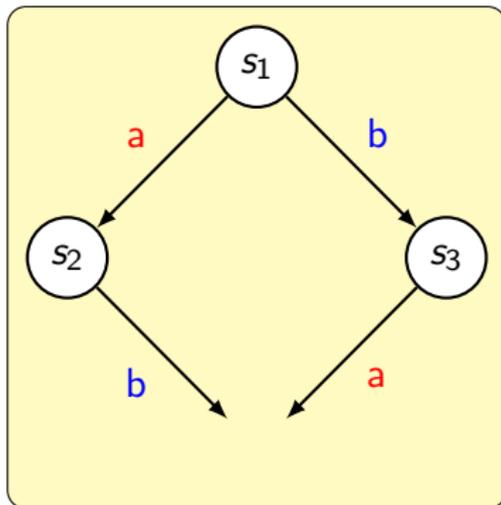


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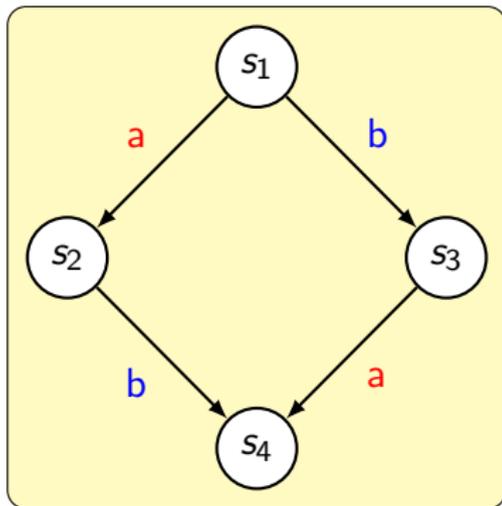


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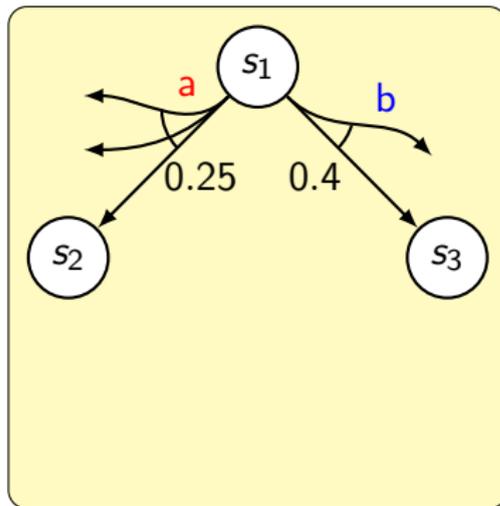
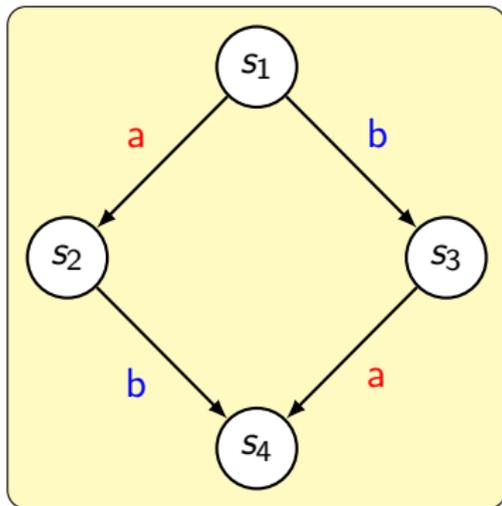


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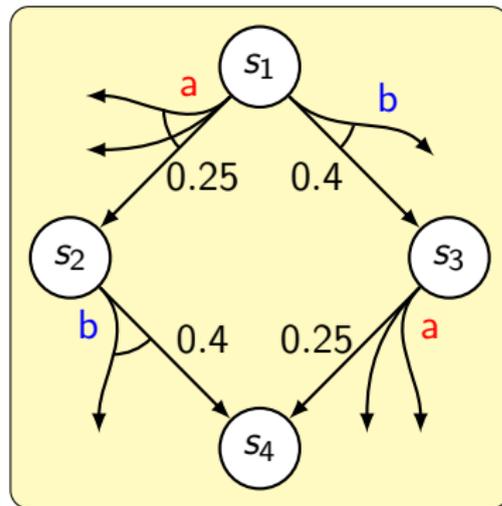
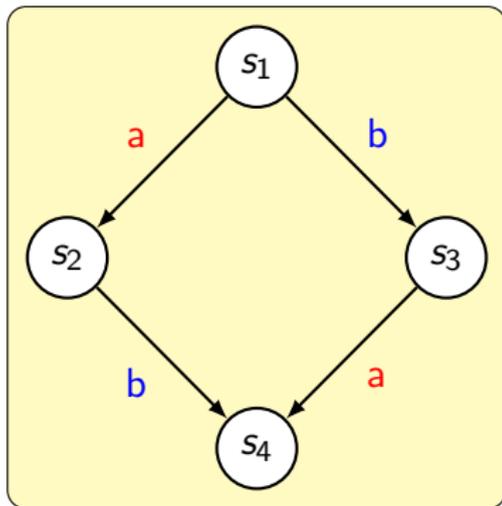


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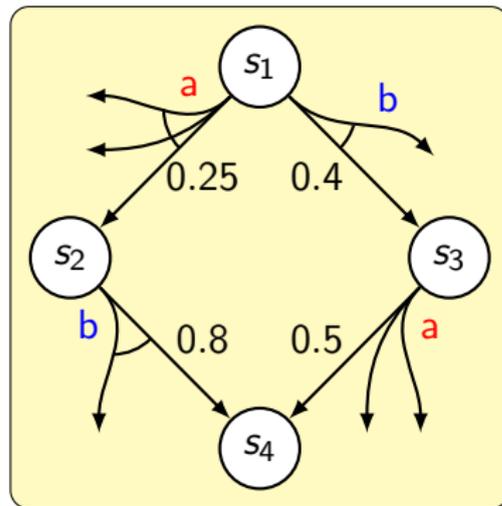
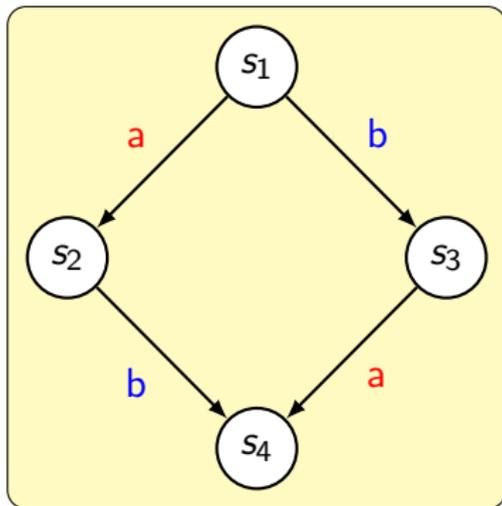


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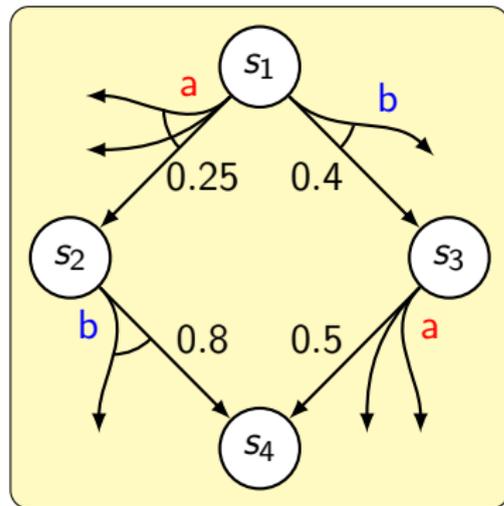
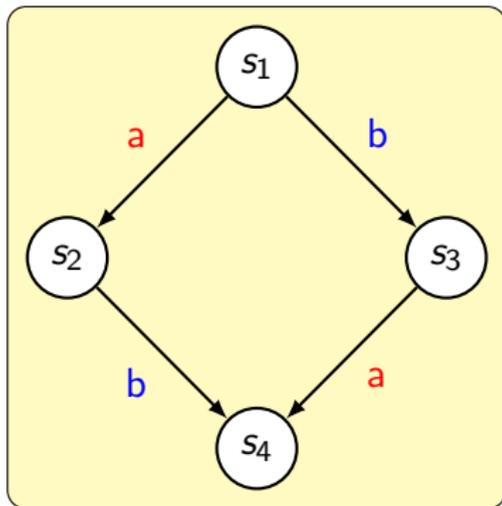


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Independence of *a* and *b*:



$$\mathbb{P}[s_1 \xrightarrow{ab} s] = \mathbb{P}[s_1 \xrightarrow{ba} s], \forall s$$

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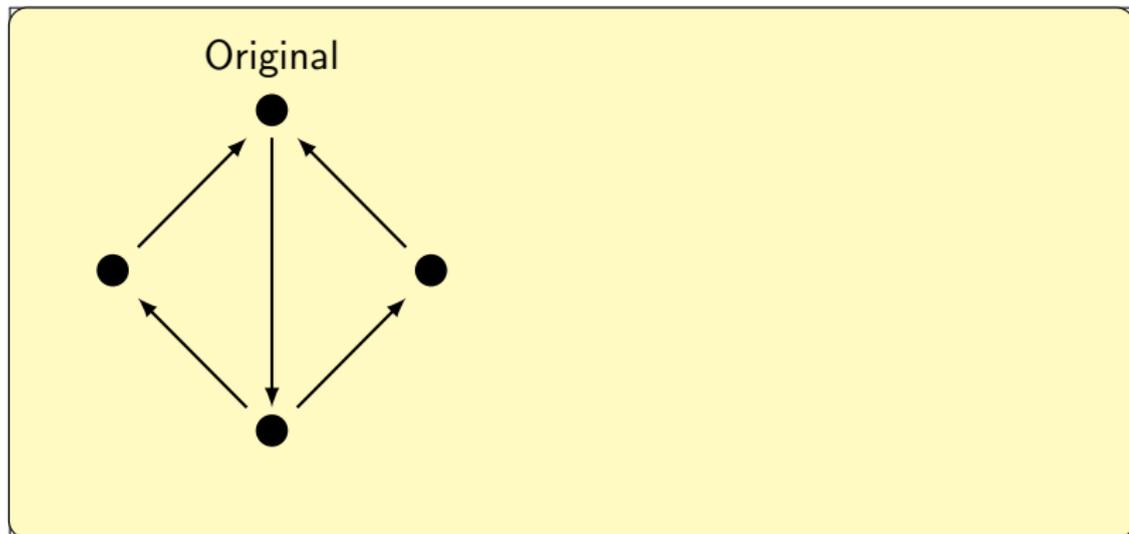
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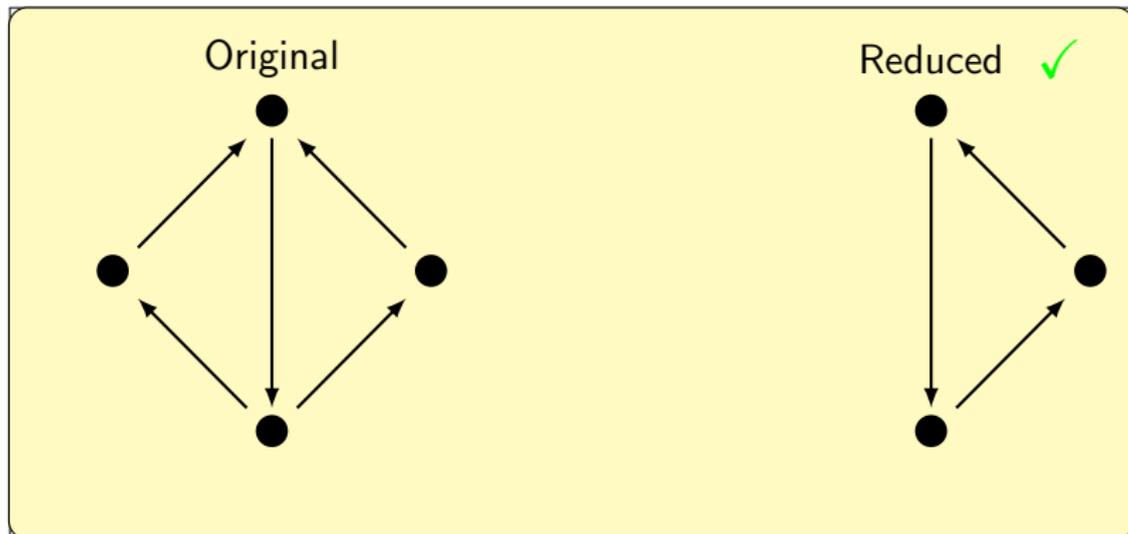


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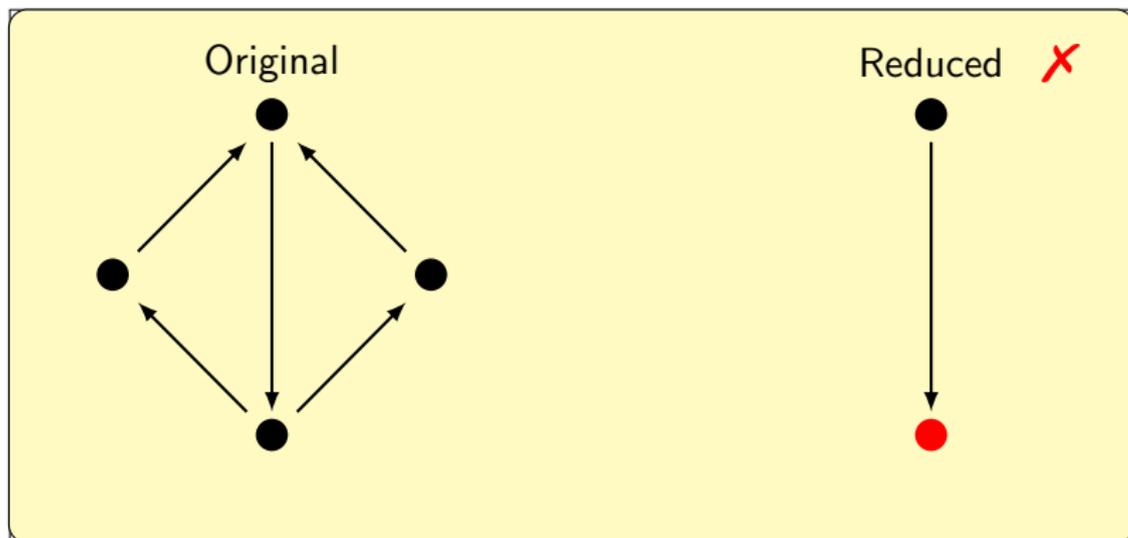


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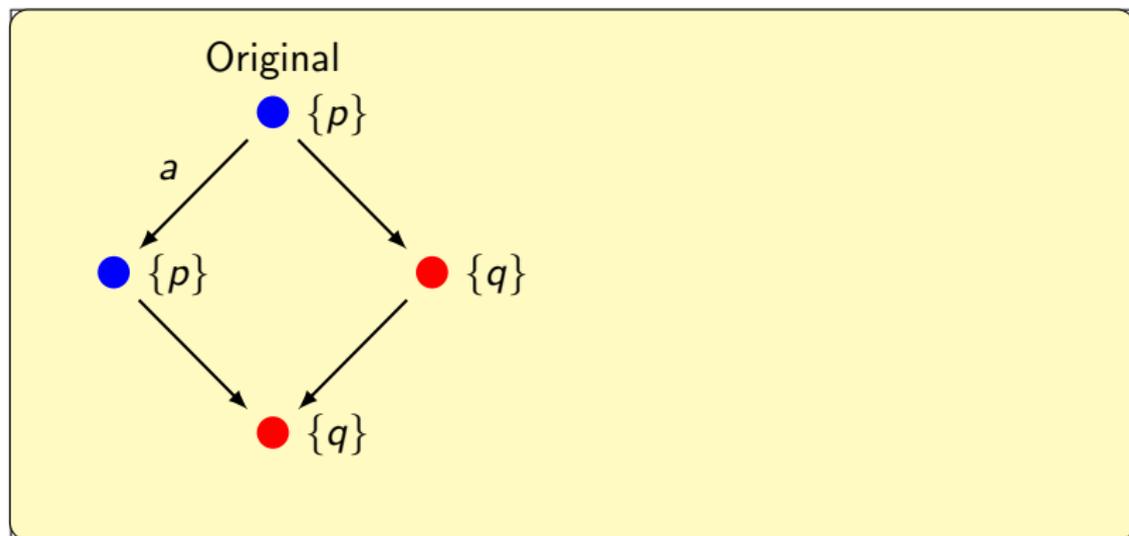
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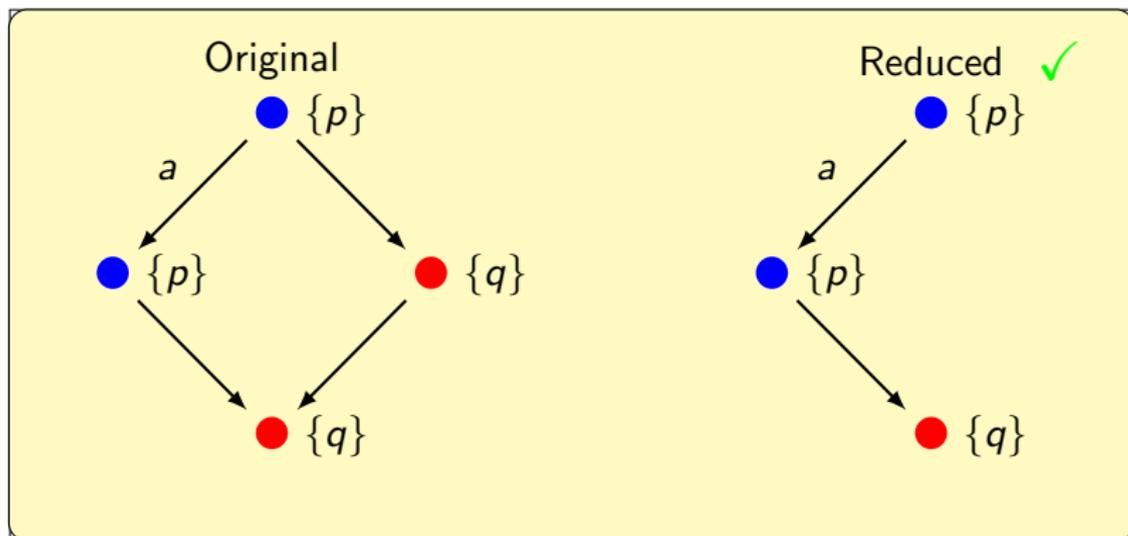


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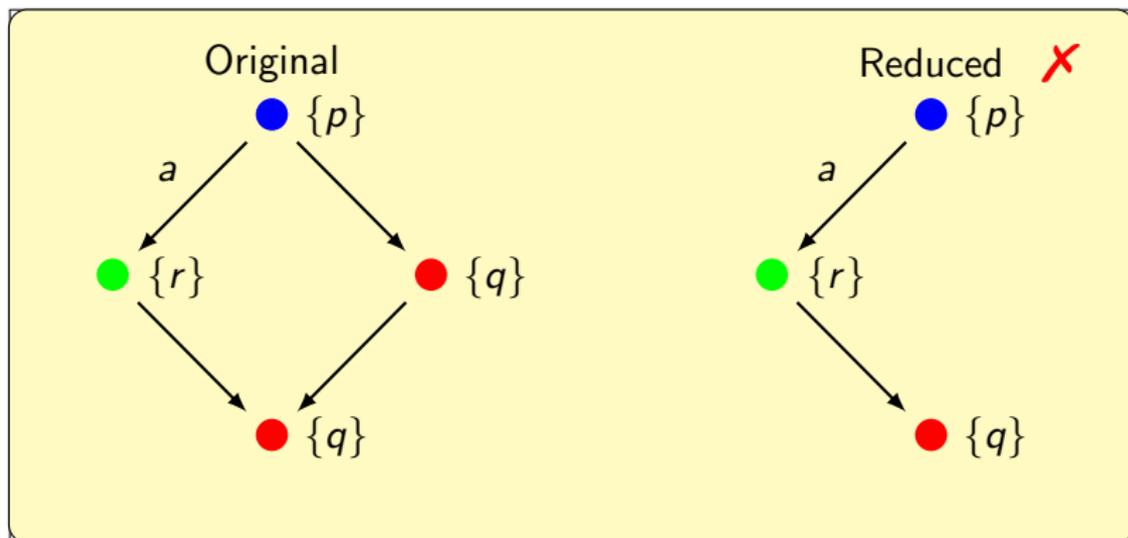


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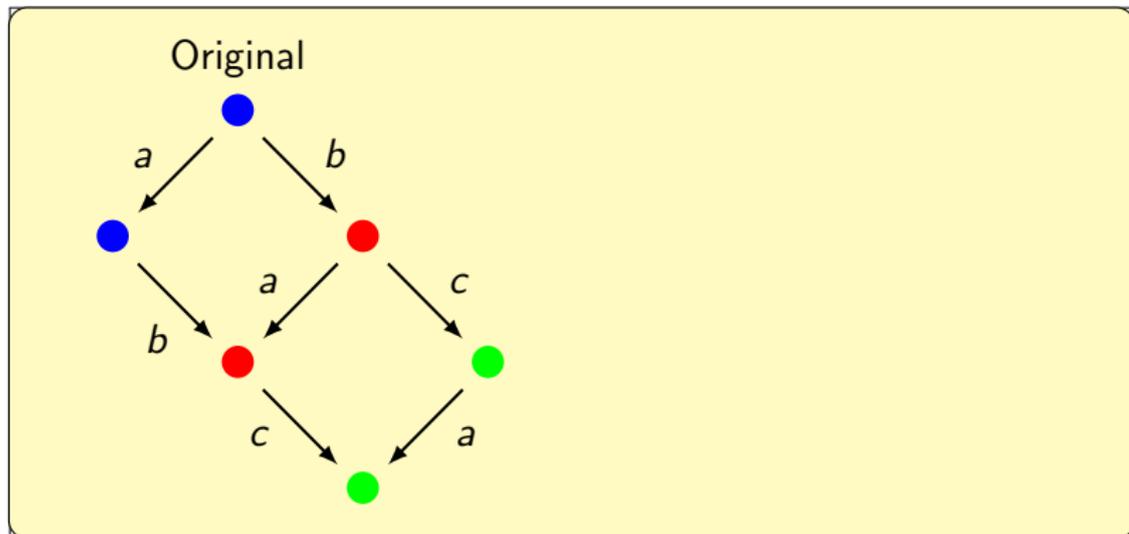
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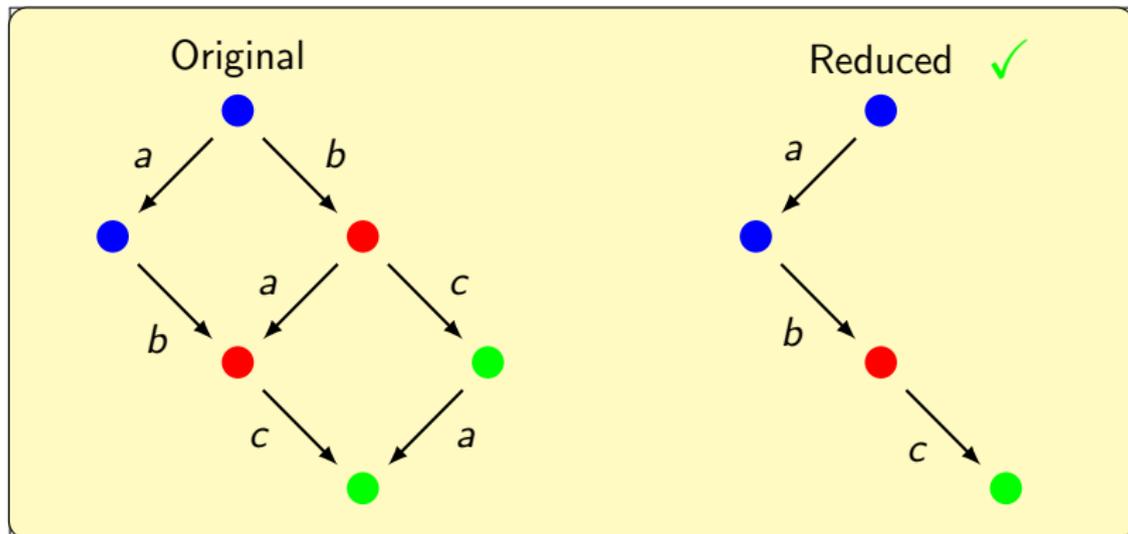


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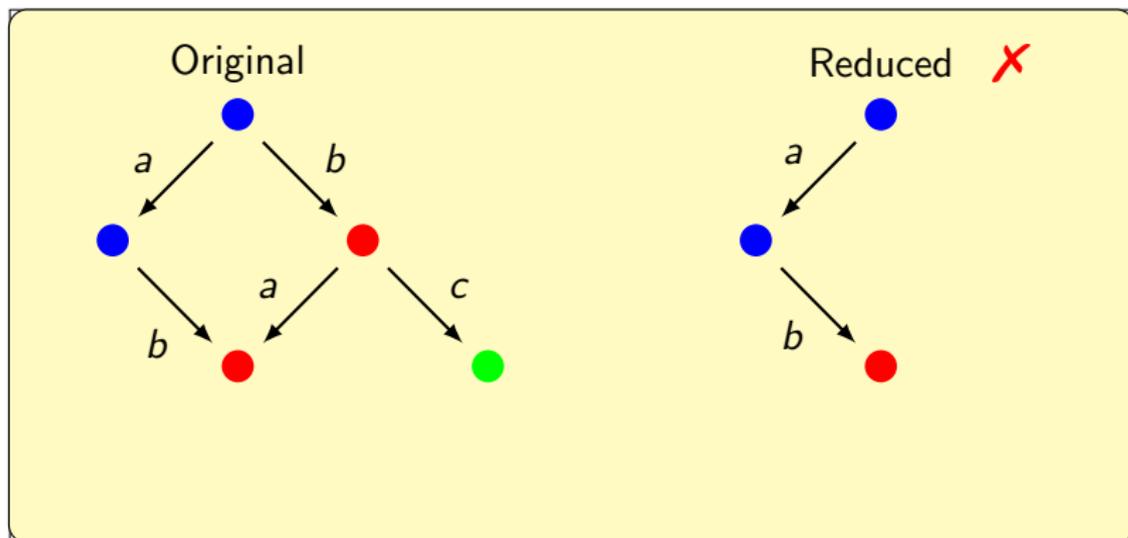


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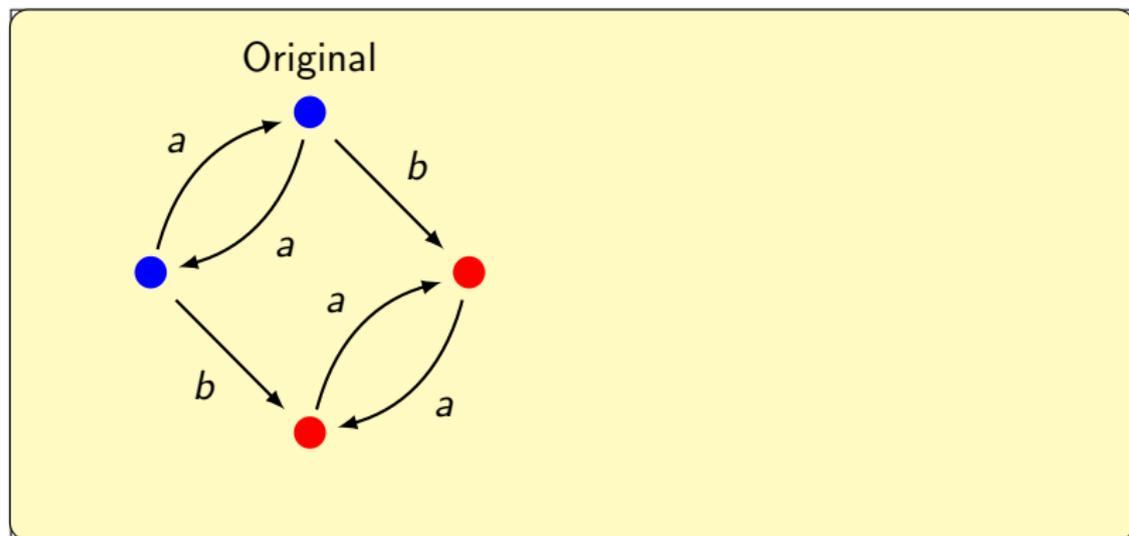
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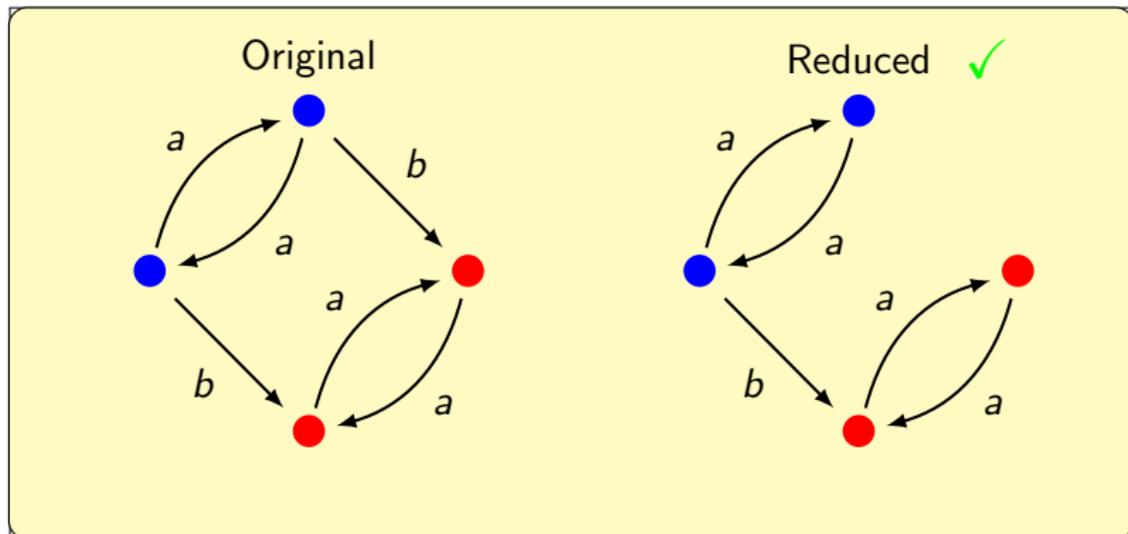


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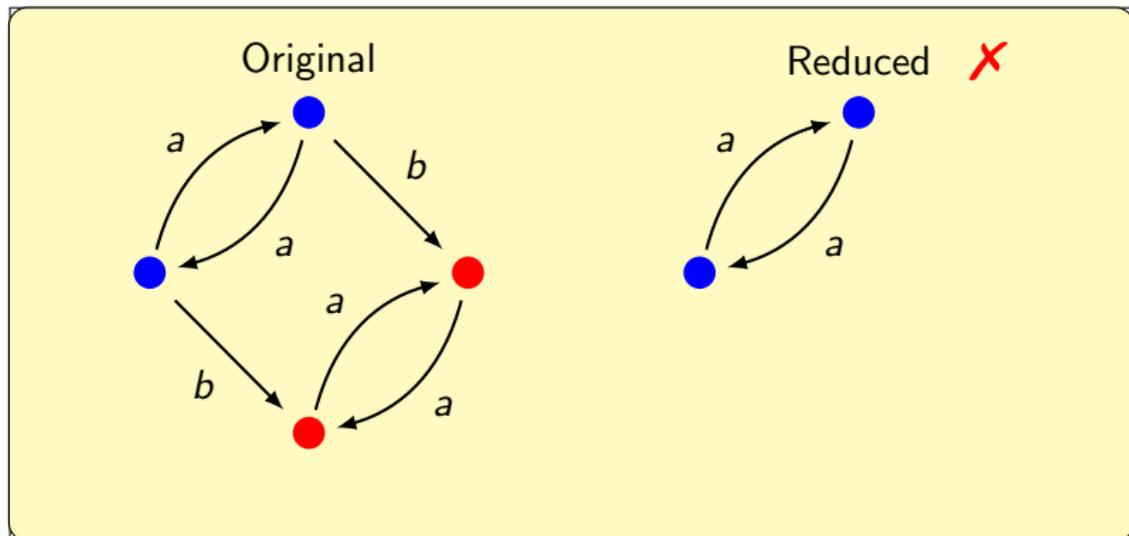


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- A4** if $R(s) \neq \text{enabled}(s)$, then $|R(s)| = 1$ and the chosen action is deterministic

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A4 if $R(s) \neq \text{enabled}(s)$, then $|R(s)| = 1$ and the chosen action is **deterministic and stuttering**

Confluence

Confluence reduction [Timmer, Stoelinga, van de Pol, 2011]

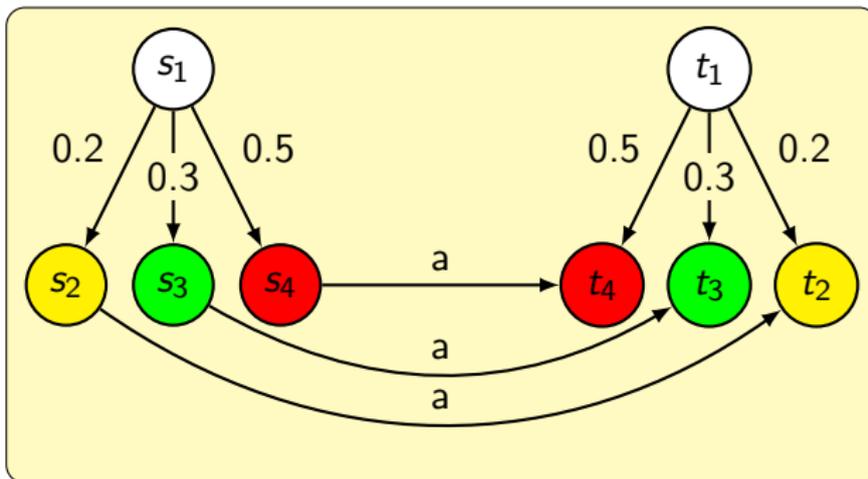
- Based on [equivalent distributions](#) and [confluent transitions](#)

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T -equivalent distributions

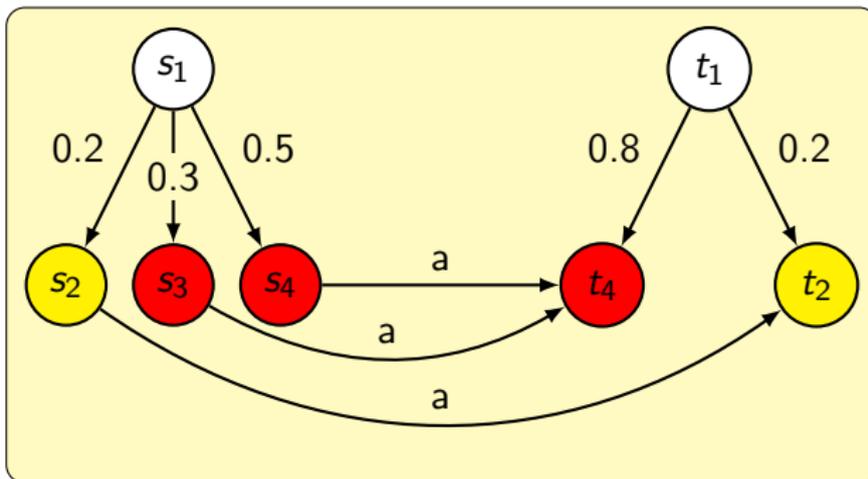


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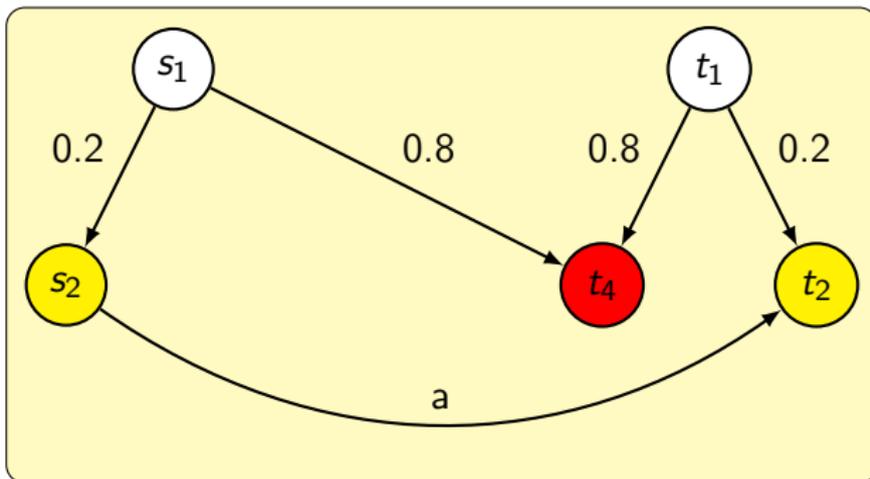


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T -equivalent distributions



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The main idea:

- Choose a set T of transitions
- Make sure all of them are **confluent**
- $R(s) = \text{enabled}(s)$ or $R(s) = \{a\}$ such that $(s \xrightarrow{a} t) \in T$

Confluence

Confluence reduction [Timmer, Stoelinga, van de Pol, 2011]

- Based on **equivalent distributions** and **confluent transitions**

The main idea:

- Choose a set T of transitions
- Make sure all of them are **confluent**
- $R(s) = \text{enabled}(s)$ or $R(s) = \{a\}$ such that $(s \xrightarrow{a} t) \in T$
- Make sure T is **acyclic** to prevent infinite postponing

Confluence

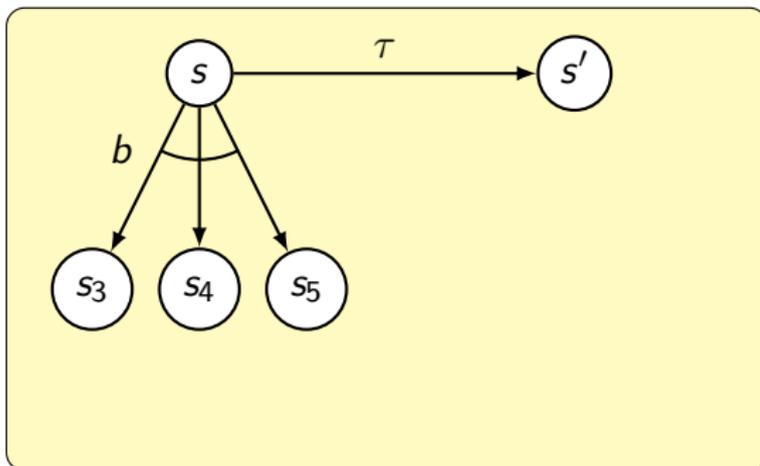
A set of transitions T is confluent if

- Every transition in T is labelled by a **deterministic stuttering** action
- If $s \xrightarrow{\tau} s' \in T$ and $s \xrightarrow{b} \mu$, then
 - 1 either $s' \xrightarrow{b} \nu$ and μ is T -equivalent to ν
 - 2 or $\mu(s') = 1$ (b deterministically goes to s')

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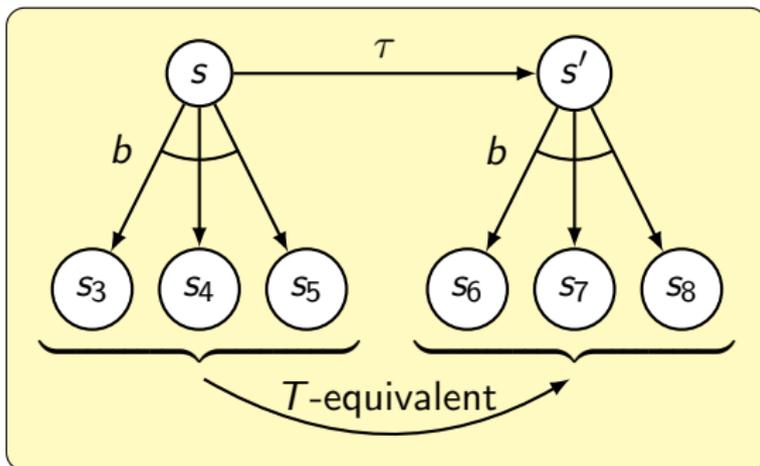
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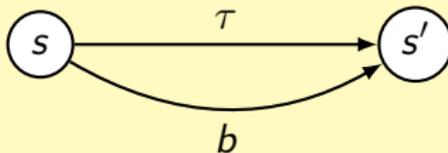
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Differences between ample sets and confluence:

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Comparison – POR implies Confluence

Theorem

*Let R be a reduction function satisfying the ample set conditions.
Then, all remaining transitions are confluent.*

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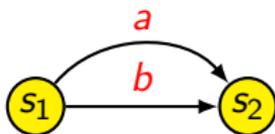
Or:

Any reduction allowed by partial-order reduction is also allowed by confluence reduction.

Proof (sketch).

- 1 Take the set of all remaining transitions of the partial-order reduction.
- 2 Recursively add transitions needed to complete the confluence diamonds.
- 3 Prove that the resulting set is indeed confluent.

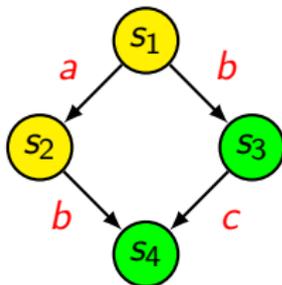
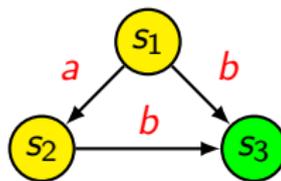
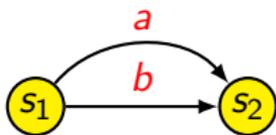
Comparison – Confluence does not imply POR



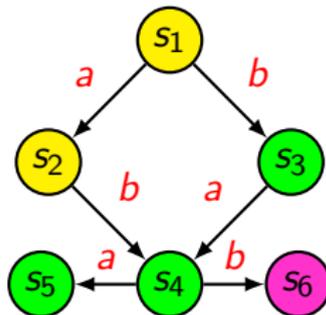
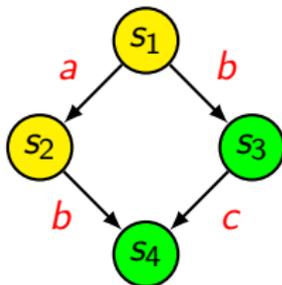
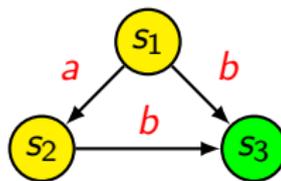
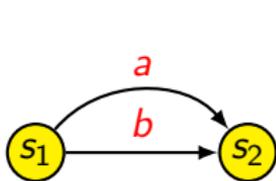
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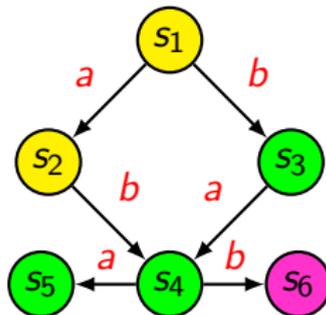
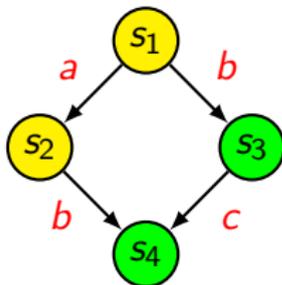
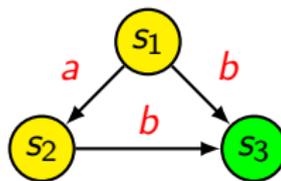
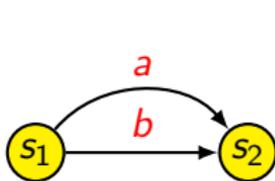
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POR's notion of independence is stronger than necessary.

Strengthening of confluence

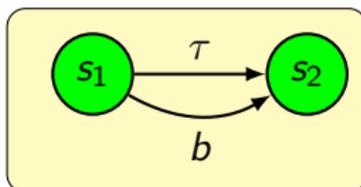
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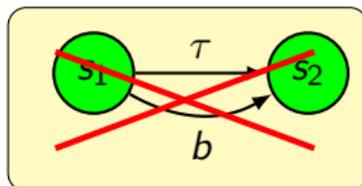
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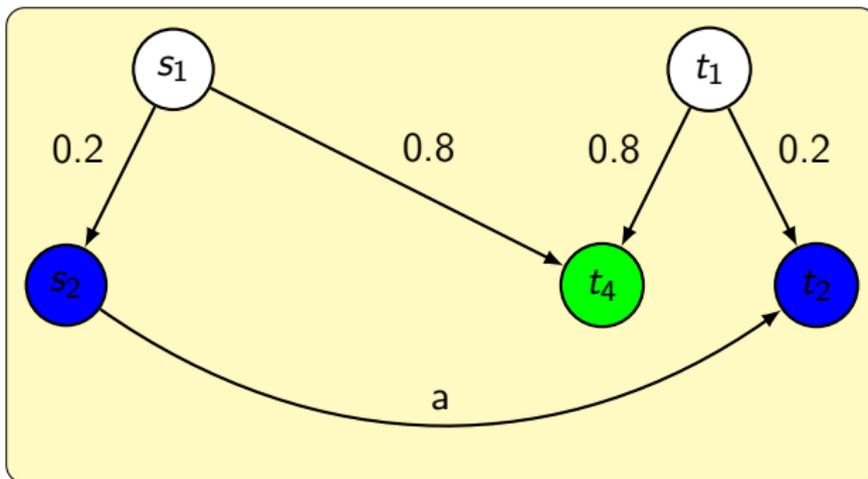
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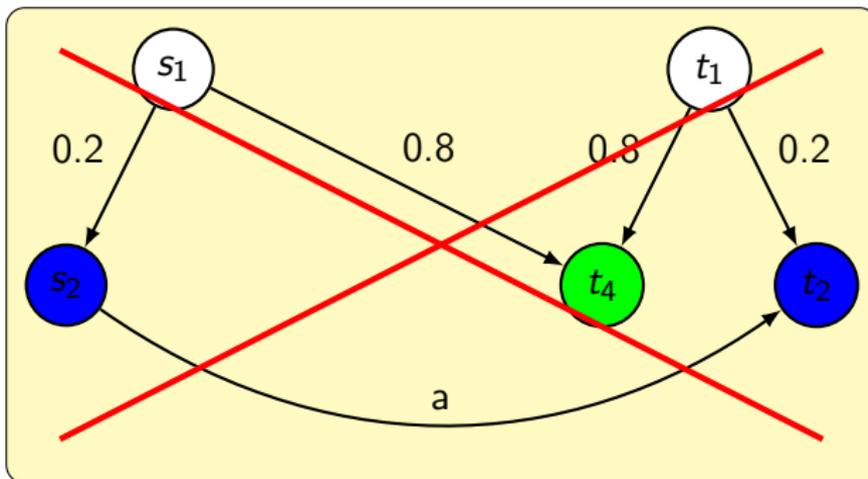
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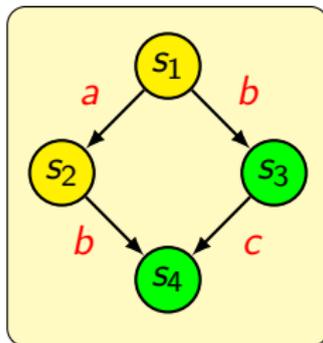
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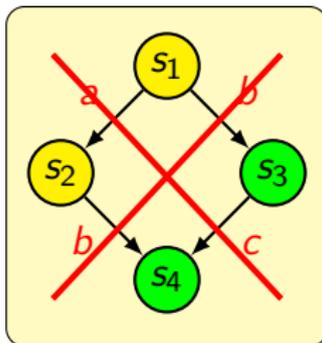
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Relaxing of partial-order reduction

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- Relax the [dependency condition](#)

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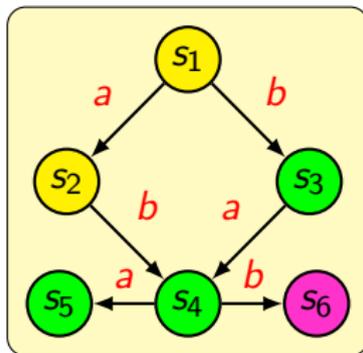
For every original path $s \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_n} s_n \xrightarrow{b} t$ such that $b \neq R(s)$ and $R(s)$ depends on b at s , there exists an i such that $a_i \in R(s)$

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Strengthening of confluence

Theorem

Every acyclic action-separable strengthened confluence reduction is a relaxed ample set reduction and vice versa.

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Corollary

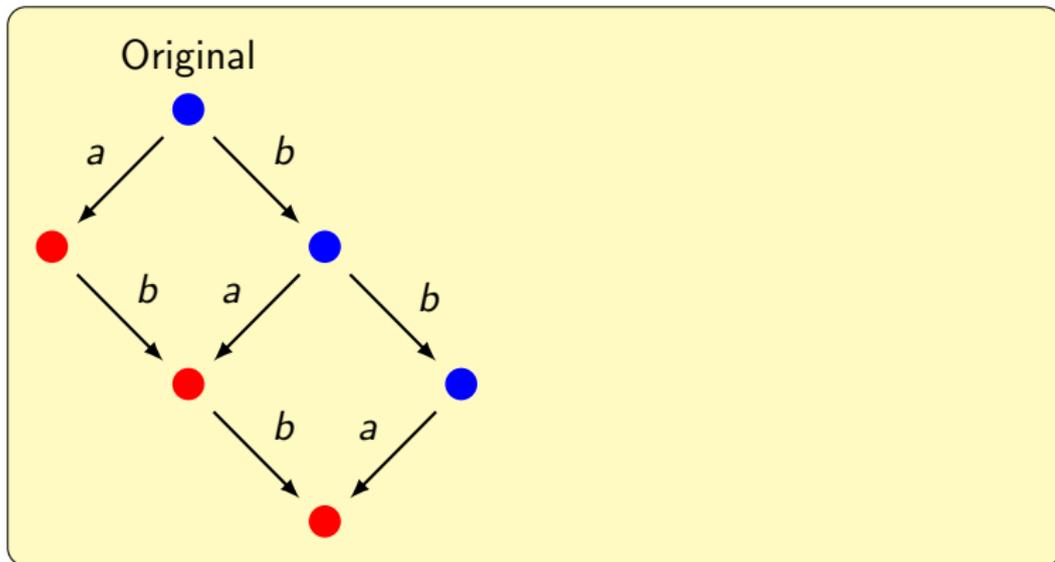
In the non-probabilistic setting, the same statements hold: confluence is stronger than partial-order reduction, and the notions are equivalent for the adjusted definitions.

Implications

State space generation using representatives:

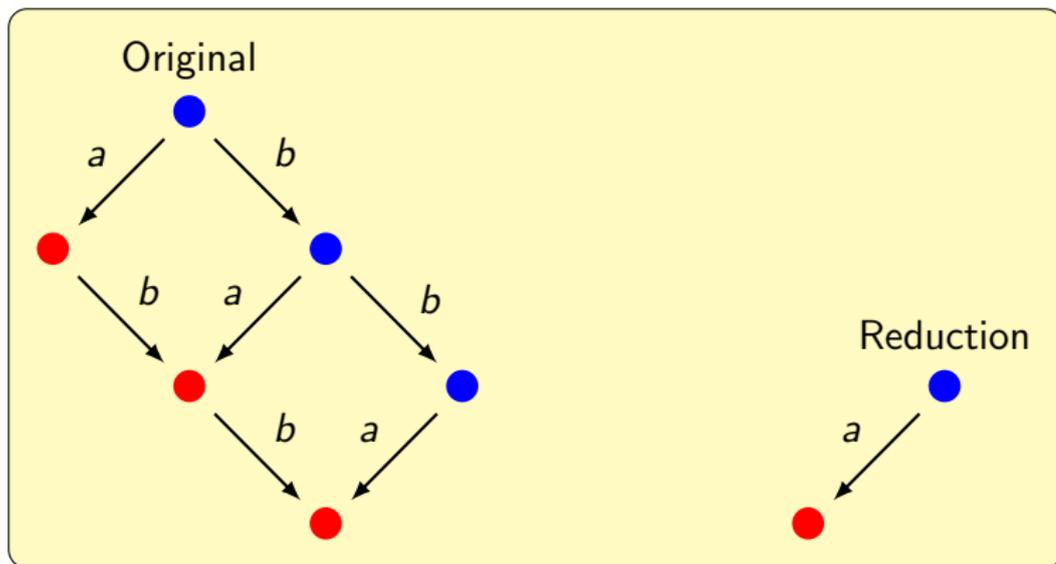
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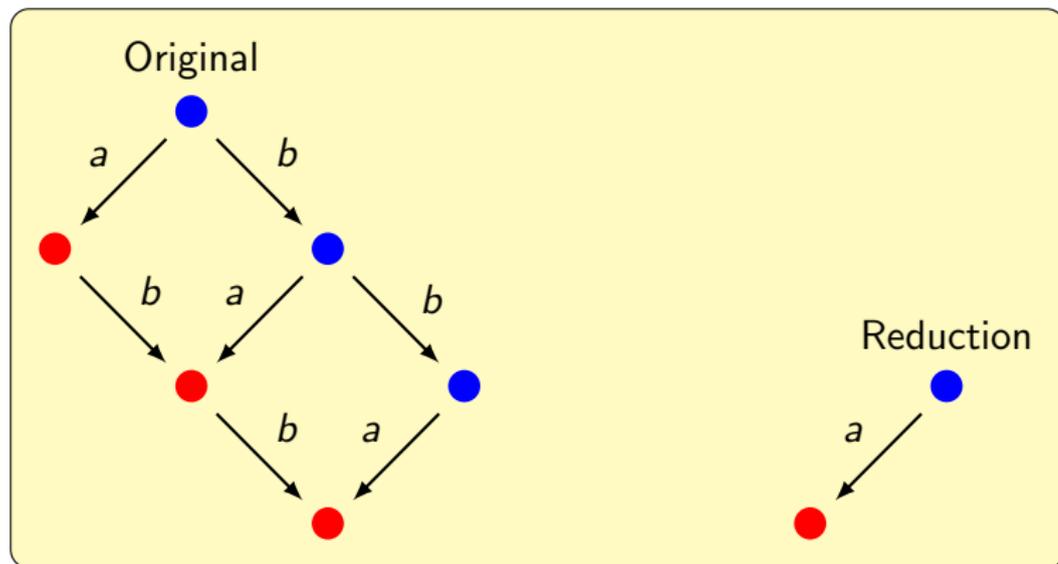
Implications

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Implications

State space generation using representatives:



- Representative in **bottom strongly connected component**
- **Additional reduction** of states and transitions
- **No need for an explicit cycle condition anymore!**

Conclusions

What to take home from this...

- We adapted the existing notion of **confluence reduction** to work in a state-based setting **with MDPs**.
- We proved that **every ample set can be mimicked by a confluent set**, but the the **converse doesn't always hold**.
- We showed how to make ample set reduction and confluence reduction **equivalent**
- We demonstrated one implication of our results, **applying a technique from confluence reduction to POR**
- The results are **independent of specific heuristics**, and also hold **non-probabilistically**

Questions

Questions?

A paper, containing all details and proofs, can be found at
<http://wwwhome.cs.utwente.nl/~timmer/research.php>