Interpreting a successful testing process: risk and actual coverage

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University of Twente

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Contents

Why testing?

- Software becomes more and more complex
- Research showed that billions can be saved by testing better
- No need for the source code (black-box perspective)

Introduction – Testing

Why testing?

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- Research showed that billions can be saved by testing better
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Model-based testing

- Precise and formal
- Automatic generation and evaluations of tests
- Repeatable and scientific basis for product testing

Introduction – Risk and coverage

Why do we need risk and coverage?

- Testing is inherently incomplete
- Testing does increase our confidence in the system
- A notion of *quality* of a test suite is necessary
- Two fundamental concepts: risk and coverage

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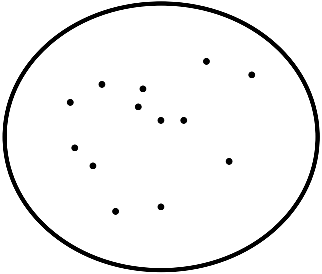

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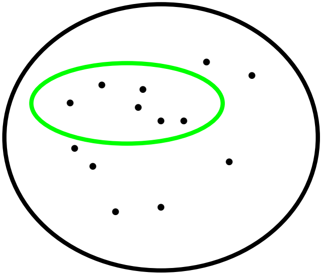

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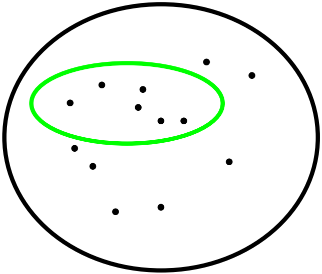

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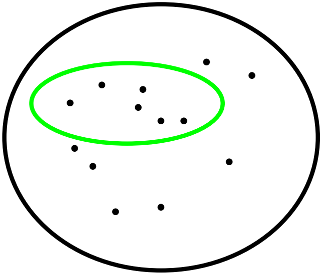

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Coverage: $\frac{6}{13} = 46\%$

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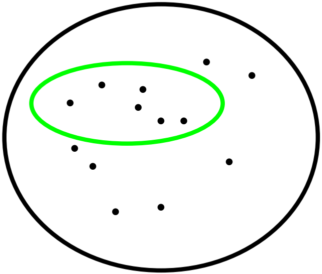

Informal calculation

Coverage: $\frac{6}{13} = 46\%$

Risk: $7 \cdot 0.1 \cdot \$10 = \$7$

Introduction – Existing approaches

Existing coverage measures

- Statement coverage
- State/transition coverage

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Limitations:

- all faults are considered of equal severity
- likely locations for fault occurrence are not taken into account
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Existing risk measures

- Bach
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Existing risk measures

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Limitations:

- Informal
- Based on heuristics
- Only identify testing order for components

Introduction – Preliminaries

Starting point: semantic coverage

Previous work by Brandán Briones, Brinksma and Stoelinga

- System considered as black box
- Semantic point of view
- Fault weights

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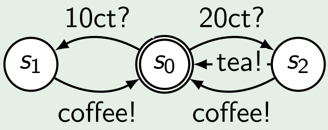
Labelled transition systems



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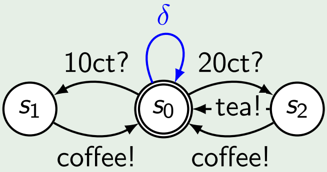

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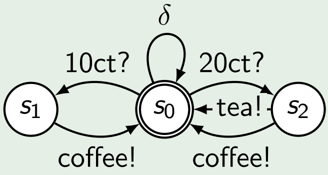

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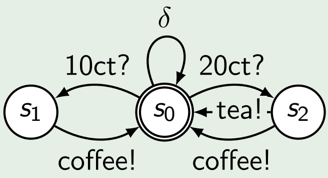

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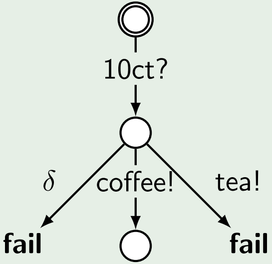
Test cases



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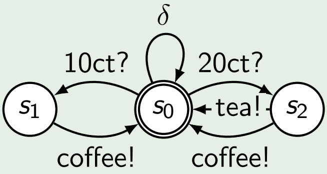

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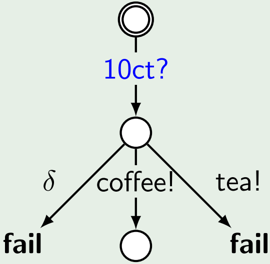

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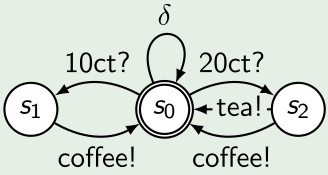

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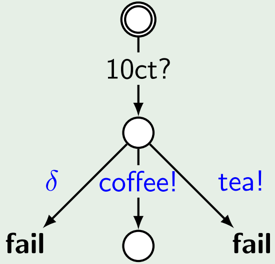

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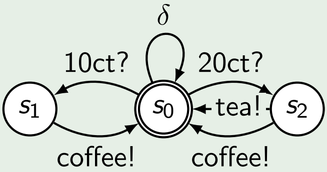

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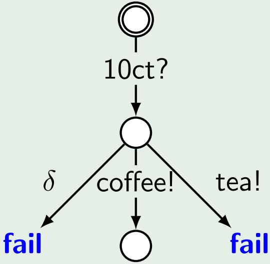

The WFS Model – Weighted Fault Specifications

Weighted fault specification

A WFS^- consists of

- An LTS (expected system behaviour)
- An error function (probability of faults)
- A weight function (severity of faults)

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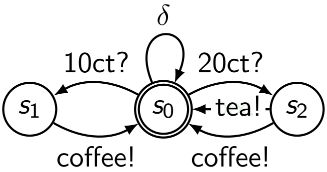

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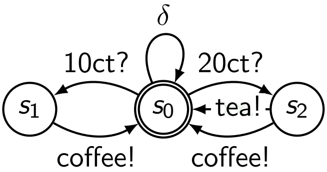

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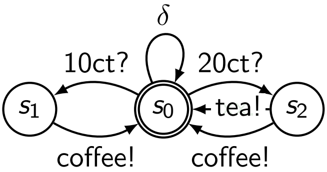

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$$w(\text{10ct?}) = 15$$
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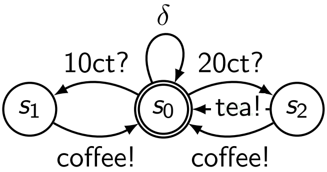

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(For more details see TechRep)

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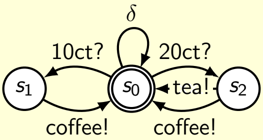

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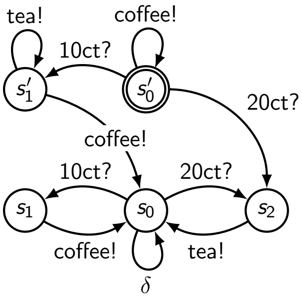

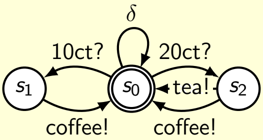

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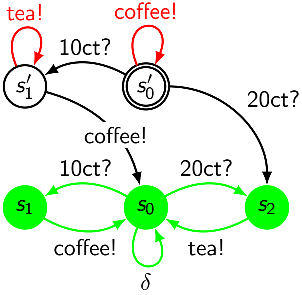

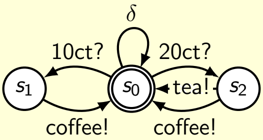

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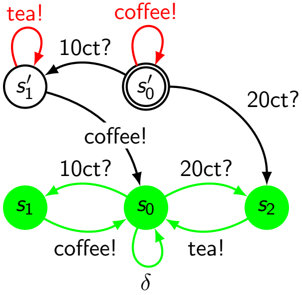

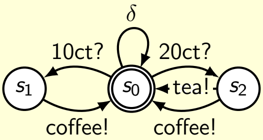

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Fault weight: $10 + 15 = 25$

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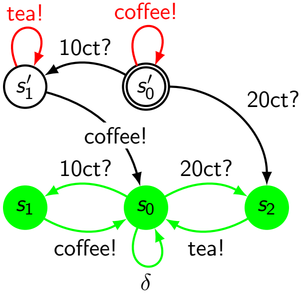

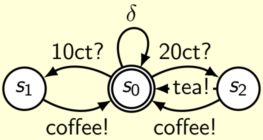

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Fault weight: $10 + 15 = 25$

(We are only interested in whether a fault can occur, not in which one)

Definition

Given a test suite T and a passing execution E, we define

$$\text{risk}(T, E) = \mathbb{E}[w(\text{Impl}) \mid \text{observe } E]$$

i.e., the fault weight still expected to be present after observing E.

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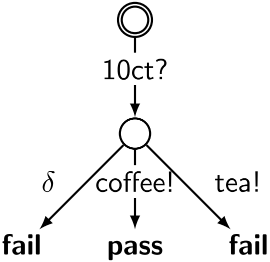
Risk


10ct?
δ
coffee!
tea!
fail
pass
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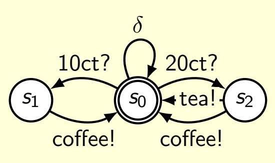
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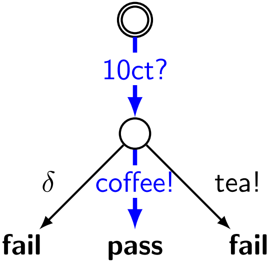
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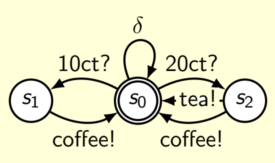
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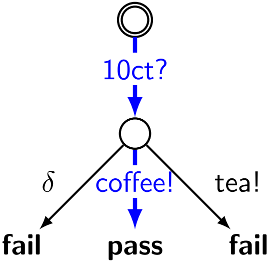
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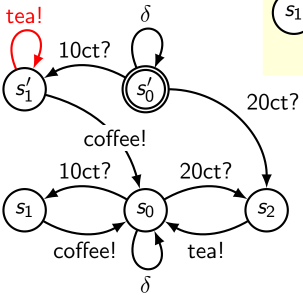
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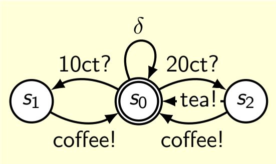
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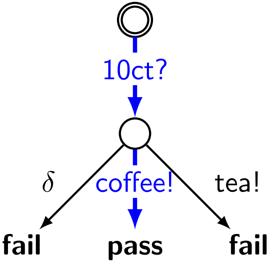
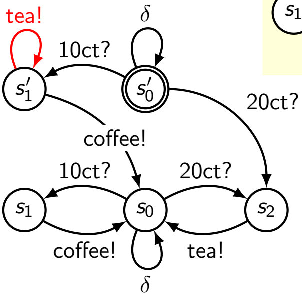
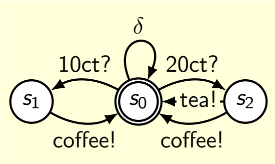
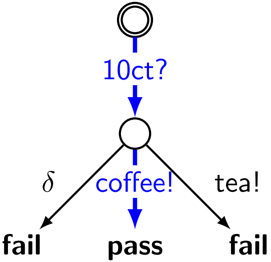
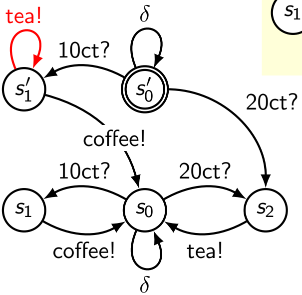


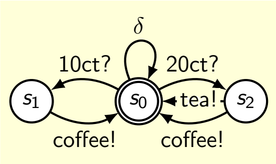




Nondeterministic output behaviour yields difficulties.







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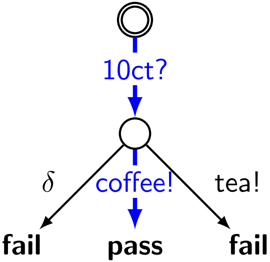
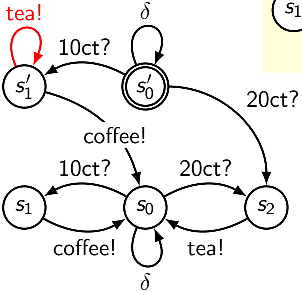
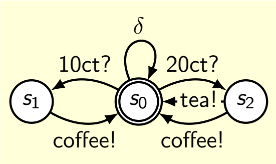
How to calculate risk (expected fault presence)?

Risk







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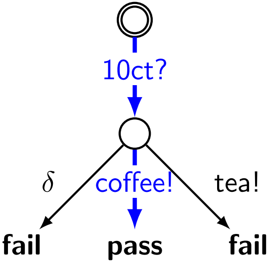

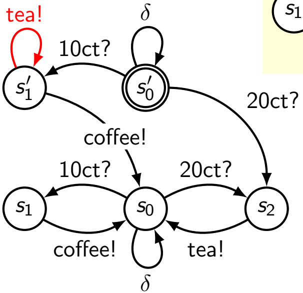

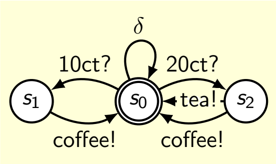

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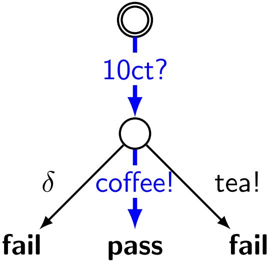
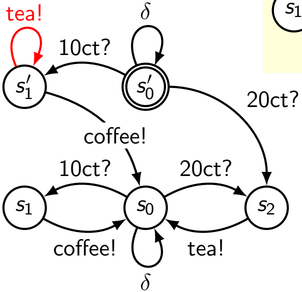
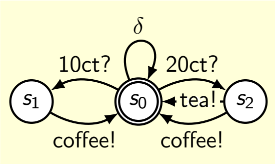
How to calculate risk (expected fault presence)?

$$\text{risk}(T, E) = \sum_{\sigma \neq 10\text{ct?}} w(\sigma) \cdot p_{\text{err}}(\sigma) + f(10\text{ct?})$$







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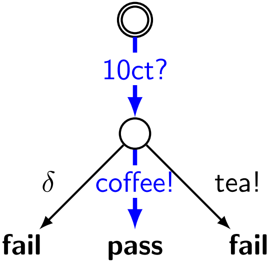
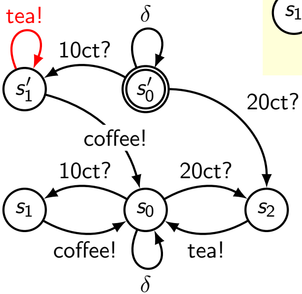
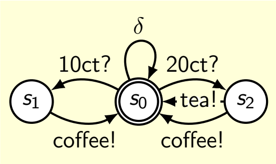
$$\text{risk}(T, E) = \sum_{\sigma \neq 10ct?} w(\sigma) \cdot p_{\text{err}}(\sigma) + f(10ct?)$$

Risk







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Weighted Fault Specifications (revisited)

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A WFS consists of

- An LTS (expected system behaviour)
- An error function (probability of faults)
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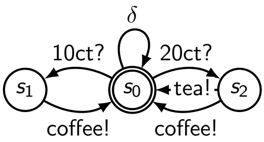

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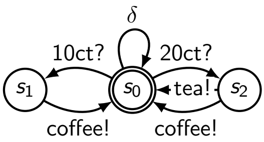

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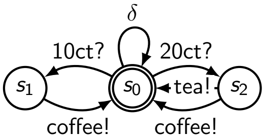

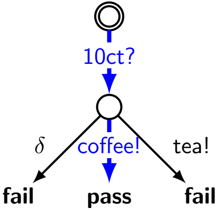

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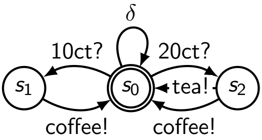

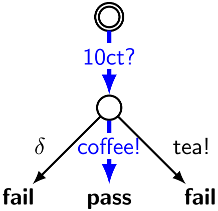

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$\mathbb{P}$[error after 10ct? | observation of E]

$= \mathbb{P}$[error after 10ct? | correct after 10ct? once]

Risk

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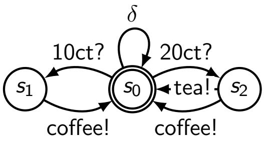

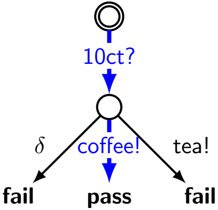

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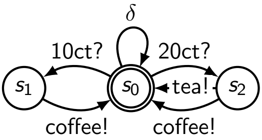

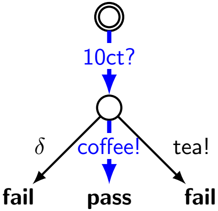

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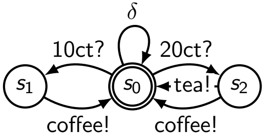

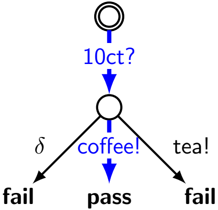

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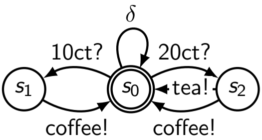

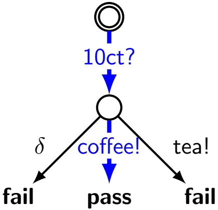

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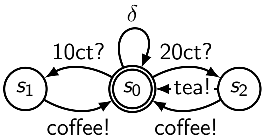

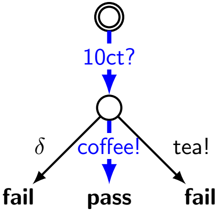

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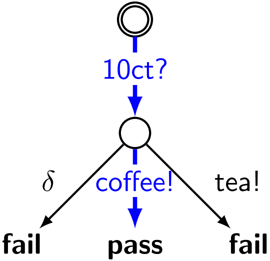

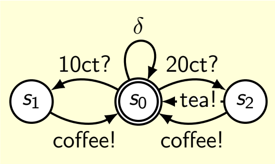

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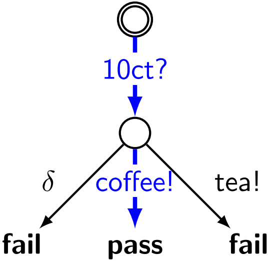

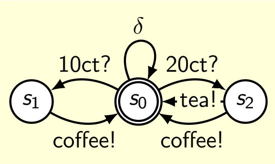

$$
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&\text{risk}(T, E) \\
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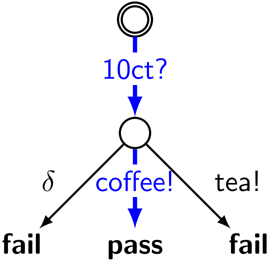

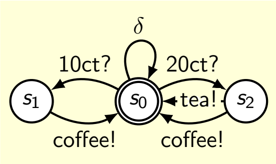

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$\text{risk}(T, E) = \mathbb{E}[w(\text{Impl}) \mid \text{observe } E]$

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$$\text{risk}(T, E) = \text{risk}(\langle\rangle, \langle\rangle) - \sum_{\sigma \in E} w(\sigma) \cdot \left(p_{\text{err}}(\sigma) - \frac{(1 - p_{\text{fail}}(\sigma))^{\text{obs}(\sigma, E)} \cdot p_{\text{err}}(\sigma)}{(1 - p_{\text{fail}}(\sigma))^{\text{obs}(\sigma, E)} \cdot p_{\text{err}}(\sigma) + 1 - p_{\text{err}}(\sigma)} \right)$$

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Although $\text{risk}(\langle\rangle, \langle\rangle) = \sum_\sigma w(\sigma) \cdot p_{\mathrm{err}}(\sigma)$ seems infinite, it can be calculated smartly:

- w defined by truncation: the sum is already finite
- w defined by discounting: system of linear equations

Other Applications

Optimisations

- Find the optimal test suite of a given size
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Actual Coverage

- Only consider the traces that were actually tested
- Use error probability reduction as coverage measure
- Methods very similar to risk

Limitations and Possibilities

Probabilities might be hard to find, but

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- We facilitate sensitivity analysis
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It looks like we need many probabilities and weights, but

- The framework can be applied at higher levels of abstraction
- Compute risk based on error / failure probabilities of modules

Conclusions and Future Work

Main results

- Formal notion of risk
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For more details, see the technical report
(`http://fmt.cs.utwente.nl/~timmer`)

Questions