Evaluating and Predicting Actual Test Coverage

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June 25, 2008

Supervised by
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Contents

Introduction

Motivation for research on testing

- Software is getting more and more complex
- Bugs cost a lot of money
- Testing is an important validation technique in software development

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- Bugs cost a lot of money
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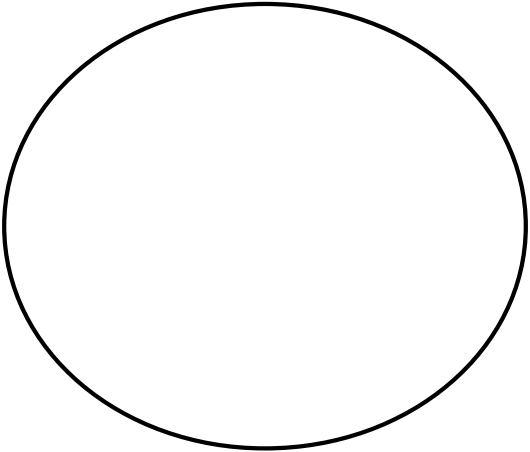
Motivation for research on test coverage

- Testing is inherently incomplete
- A notion of *quality* of a test suite is necessary
- Quantitative evaluation: how good is a test suite?
- ‘Amount’ of specification / implementation examined by a test suite

Intuition about coverage



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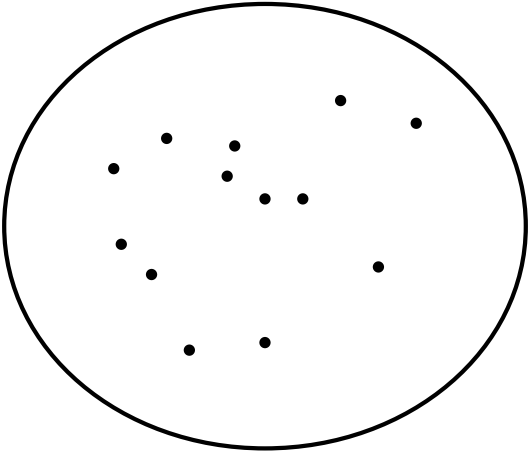

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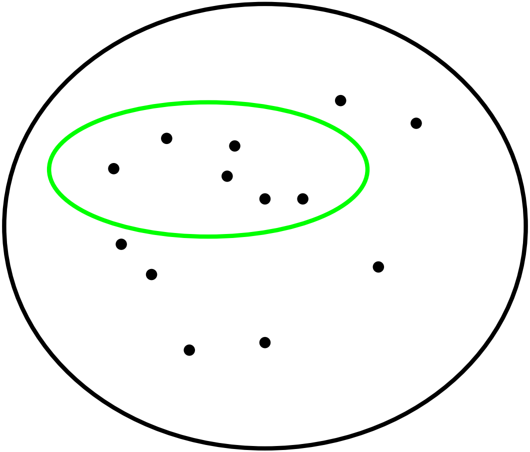

Existing approaches to coverage

Early work on coverage: code coverage

- Statement coverage
- Condition coverage
- Path coverage

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Disadvantages:
- all faults are considered of equal severity
- syntactic point of view
- different implementation, different coverage

Syntactic vs. semantic point of view

Recipe 1: vegetable soup

- Chop an union
- Slice a few carrots and a mushroom
- Boil one liter of water
- Add some Maggi
- Put everything in the water
- Wait a while

Syntactic vs. semantic point of view

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Quality: $\frac{5}{6} \cdot 10 = 8.33$

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Semantic point of view: how does it taste

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- Statement coverage
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- Path coverage

Disadvantages: - all faults are considered of equal severity
- syntactic point of view
- different implementation, different coverage

Starting point for my work: semantic coverage

Previous work by Brandán Briones, Brinksma and Stoelinga

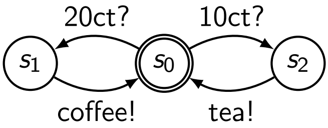
- System considered as black box
- Semantic point of view
- Error weights

Preliminaries – Labeled transition systems (LTSs)

Definition LTSs

An LTS is a tuple $\mathcal{A} = \langle S, s^0, L, \Delta \rangle$, with

- S a set of states
- s^0 the initial state
- L a set of actions (partitioned into *input* and *output* actions)
- Δ the transition relation (assumed deterministic)



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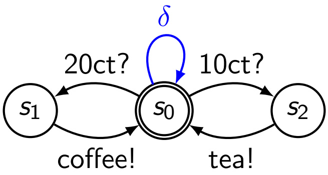
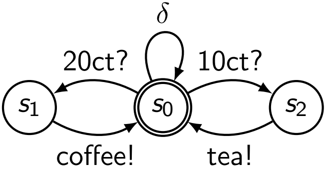


Preliminaries – Test cases for LTSs

Specification:



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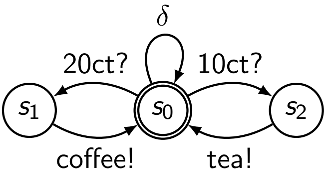
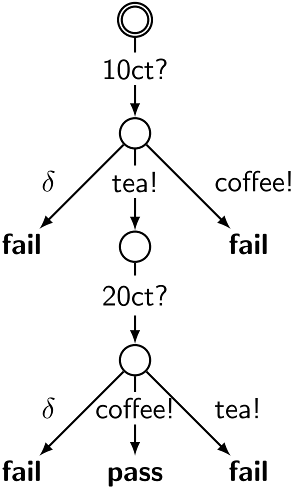


Test case:



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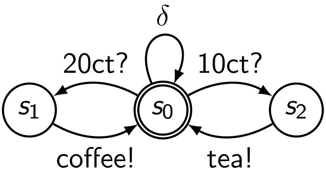
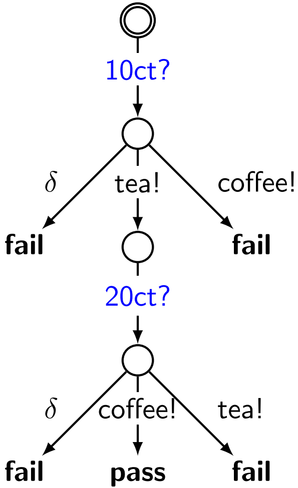


- Perform an input

Test case:



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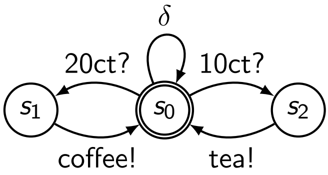
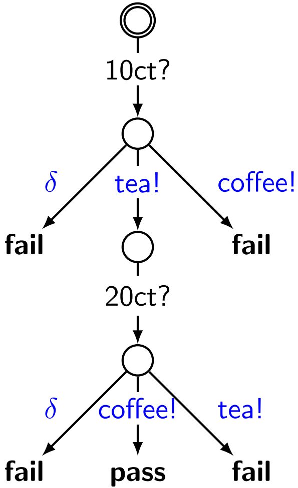


- Perform an input
- Observe all outputs

Test case:



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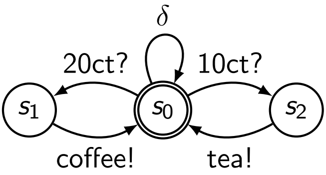
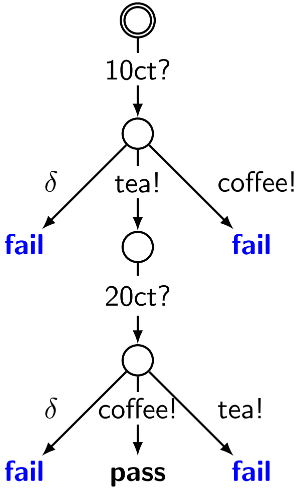
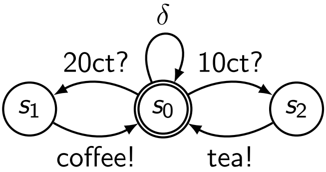


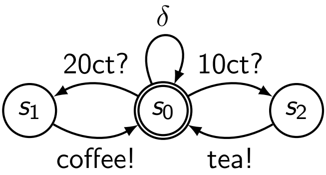
- Perform an input
- Observe all outputs
- Always stop after an error

Test case:



Preliminaries – Weighted fault models (WFMs)





$$f(coffee!) = 10$$

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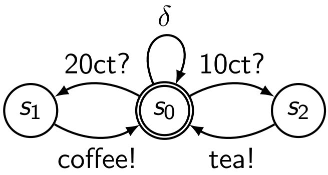

$$
\begin{aligned}
f(\mathit{coffee!}) &= 10\\
f(\mathit{10ct?\ tea!}) &= 0
\end{aligned}
$$

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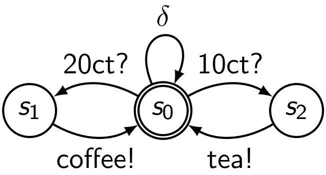
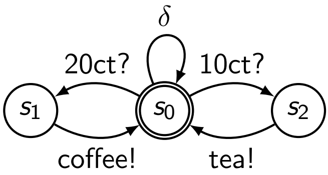


$$f(\textit{coffee!}) = 10$$
$$f(\textit{10ct? tea!}) = 0$$
$$f(\textit{10ct? coffee!}) = 5$$
$$f(\textit{10ct? } \delta) = 4$$



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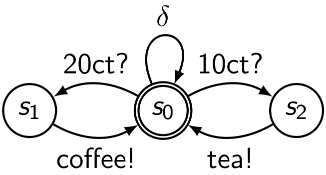

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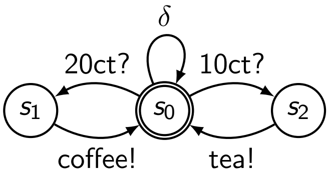

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Restriction on weighted fault models

$$0 < \sum_{\sigma \in L^*} f(\sigma) < \infty$$

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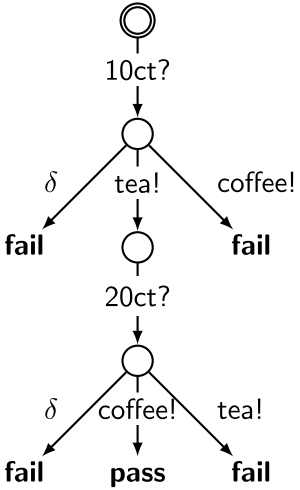

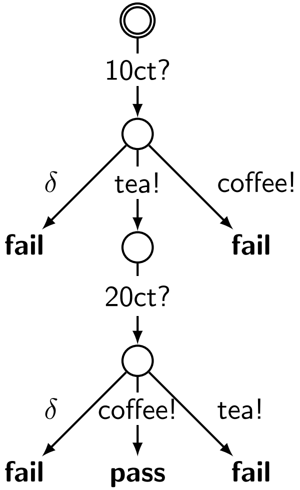

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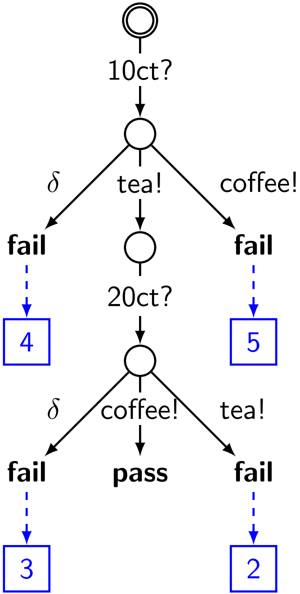

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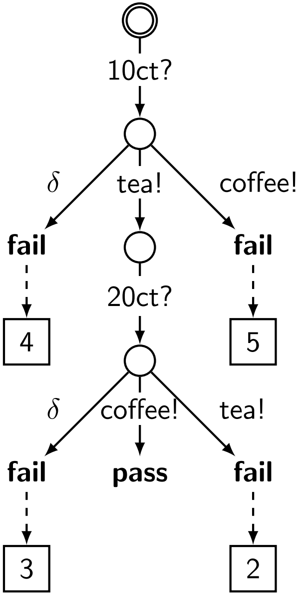
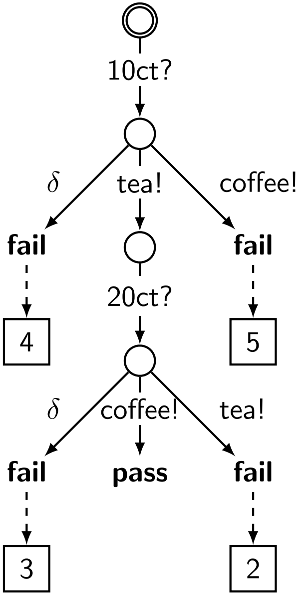


If

$$totCov = 150$$

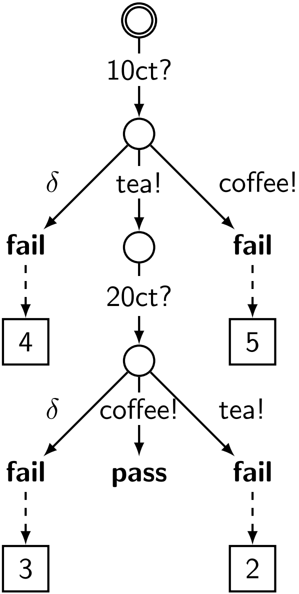


If

$$totCov = 150$$

then

$$absPotCov = 4 + 5 + 3 + 2 = 14$$



If

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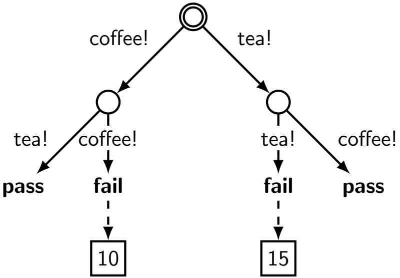
$$absPotCov = 4 + 5 + 3 + 2 = 14$$

$$relPotCov = \frac{14}{150} = 0.09$$

Limitations of potential coverage



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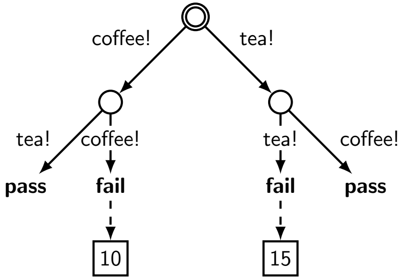
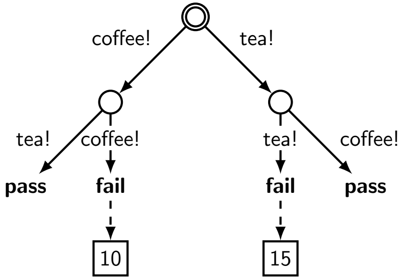


Potential coverage

$$absPotCov(f, t) = 10 + 15 = 25$$



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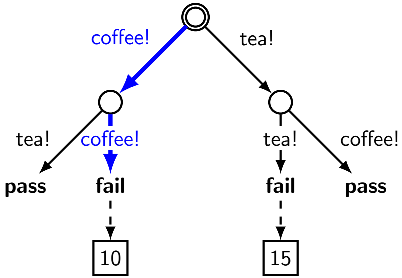
Actual coverage

- Not all faults can be detected at once
- Single executions cover only some faults
- Executing more often could increase coverage
- How many executions are needed?
- Necessary to include probabilities!

Limitations of potential coverage



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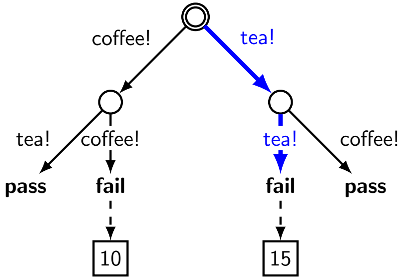

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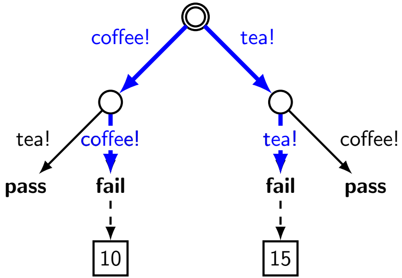

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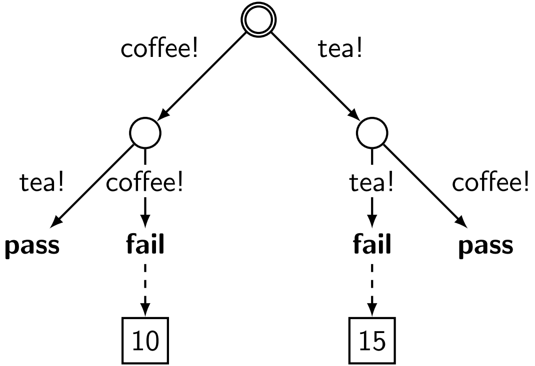
3. Predicting actual coverage:

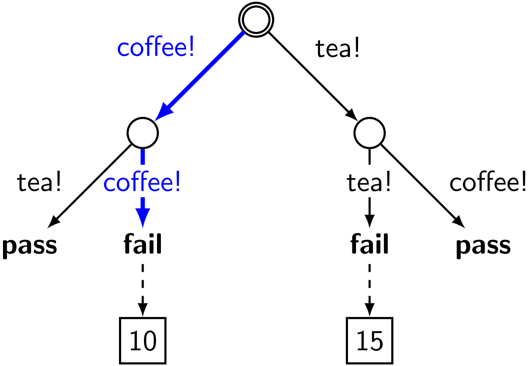
   Predicting the actual coverage a test case or test suite yields.

Fault coverage



Fault coverage

A fault is *covered* by an execution if the execution gives us information about whether the fault is present or absent



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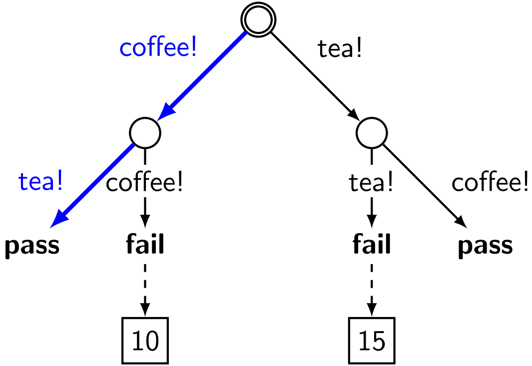

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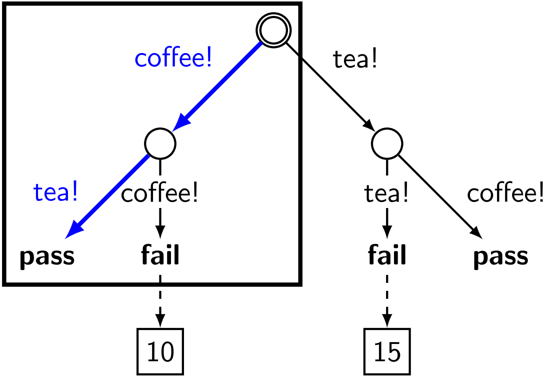

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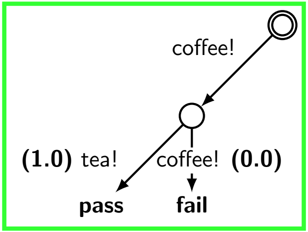
Conditional branching probabilities



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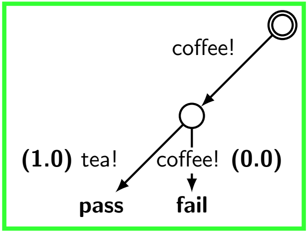

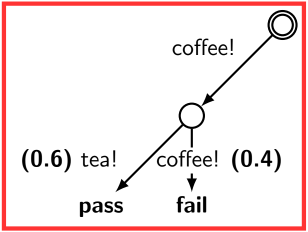

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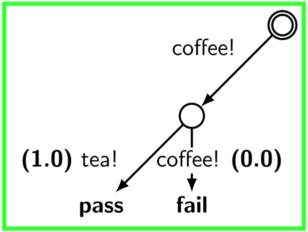

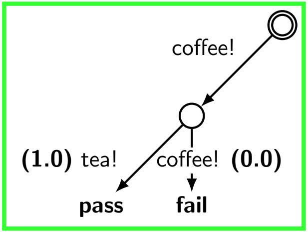

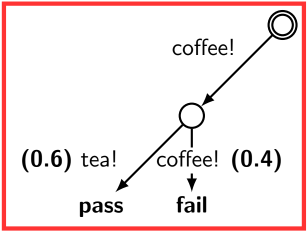

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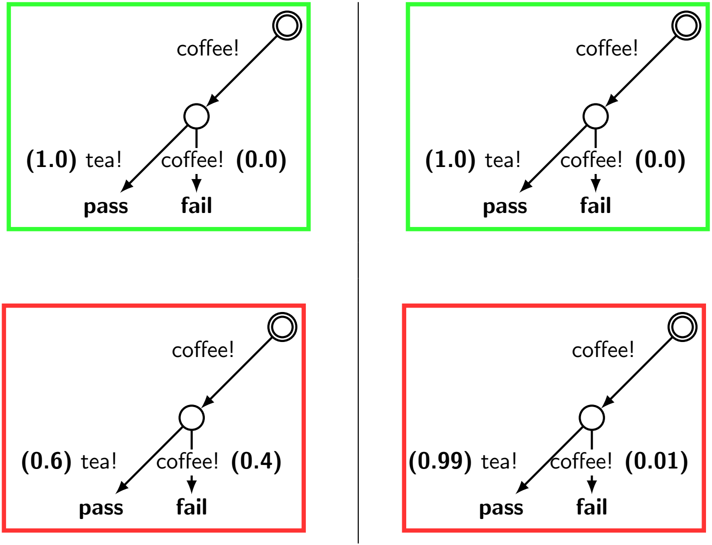

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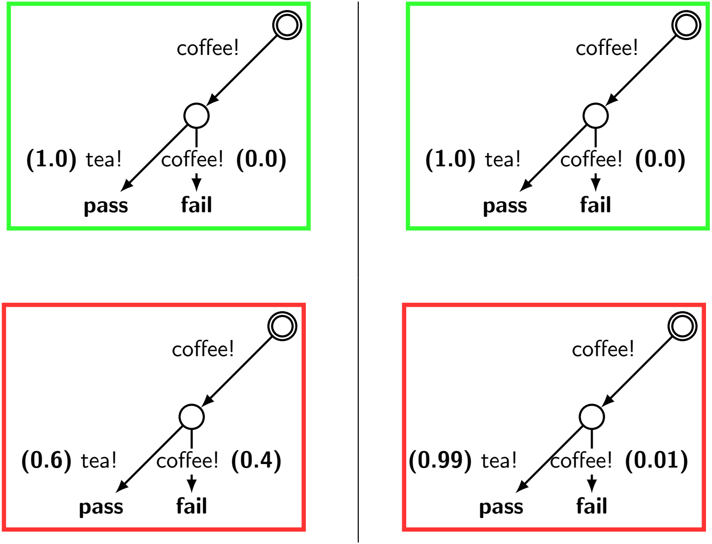
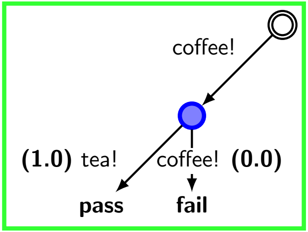
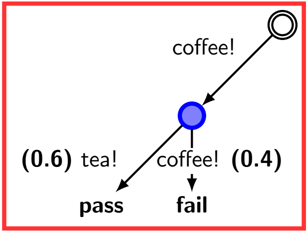


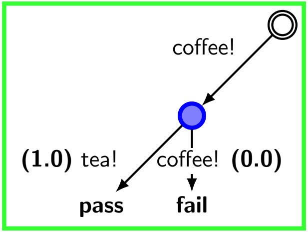
Conditional branching probabilities p^{cbr}

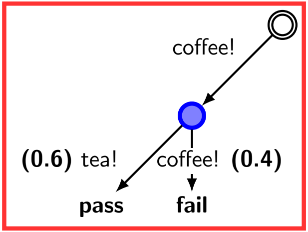




- Conditional branching probability: 0.4 ($\mathbb{P}$[*coffee! produced from blue* | *coffee! possible from blue*])

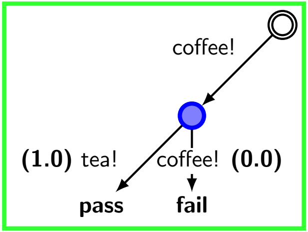
Coverage probabilities

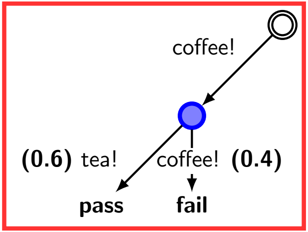




- Conditional branching probability: 0.4 ($\mathbb{P}$[*coffee! produced from blue* | *coffee! possible from blue*])
- Observation: 5 times *coffee! tea!*

Coverage probabilities



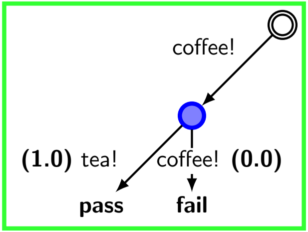


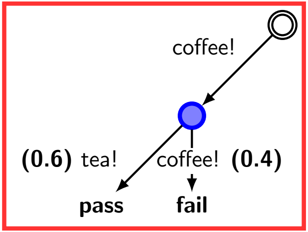
- Conditional branching probability: 0.4 ($\mathbb{P}$[*coffee! produced from blue* | *coffee! possible from blue*])

- Observation: 5 times *coffee! tea!*
- Probability of not even one *coffee!*:

$$(1 - p^{cbr})^5 = 0.6^5 = 0.08$$

Coverage probabilities





- Conditional branching probability: 0.4 ($\mathbb{P}$[*coffee! produced from blue* | *coffee! possible from blue*])

- Observation: 5 times *coffee! tea!*
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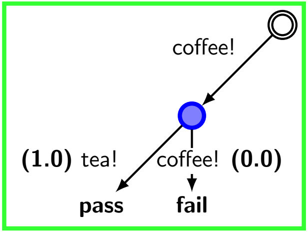
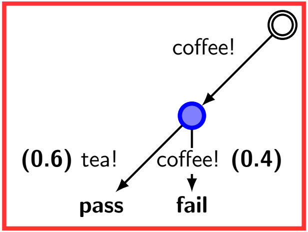
$$(1 - p^{cbr})^5 = 0.6^5 = 0.08$$

- Probability of at least one *coffee!*:

$$1 - (1 - p^{cbr})^5 = 1 - 0.6^5 = 0.92$$

Coverage probabilities





- Conditional branching probability: 0.4 ($\mathbb{P}$[*coffee! produced from blue* | *coffee! possible from blue*])
- Observation: 5 times *coffee! tea!*
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$$(1 - p^{cbr})^5 = 0.6^5 = 0.08$$

- Probability of at least one *coffee!*:

$$1 - (1 - p^{cbr})^5 = 1 - 0.6^5 = 0.92$$

- Coverage probability:

$$p^{cov} = 1 - (1 - p^{cbr})^k$$

Fault coverage

1. If a fault is shown present, it is *completely covered*
2. If a fault is shown absent, it is *partially covered*. The coverage probability denotes the fraction.

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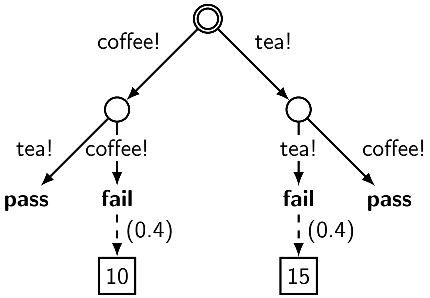

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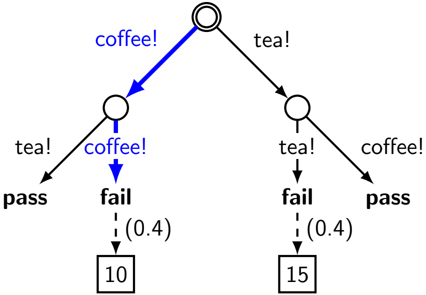

Fault coverage *coffee! coffee!*
10

Fault coverage *tea! tea!*
0

Fault coverage

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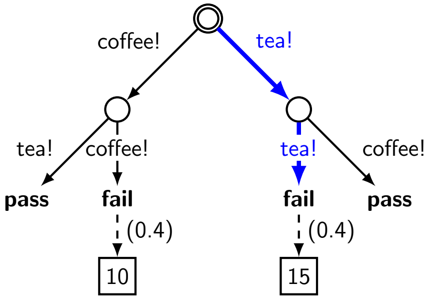


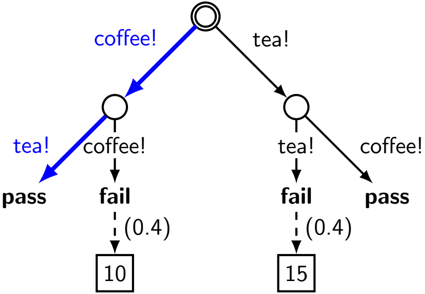
Fault coverage *coffee! coffee!*
0

Fault coverage *tea! tea!*
15

Fault coverage

Fault coverage

1. If a fault is shown present, it is *completely covered*
2. If a fault is shown absent, it is *partially covered*. The coverage probability denotes the fraction.



$$p^{cov}(\textit{coffee! coffee!})$$
$$1-(1-0.4)^3 = 0.78$$

Fault coverage *coffee! coffee!*
7.8

Fault coverage *tea! tea!*
0

Actual coverage

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Actual coverage of an execution or sequence of executions:
The sum of all fault coverages

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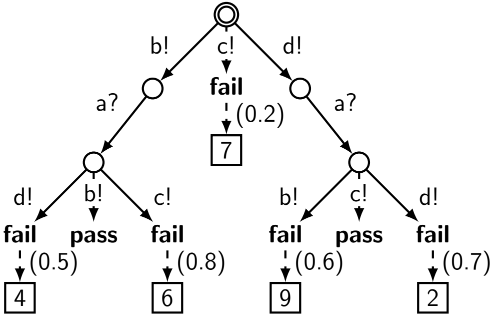

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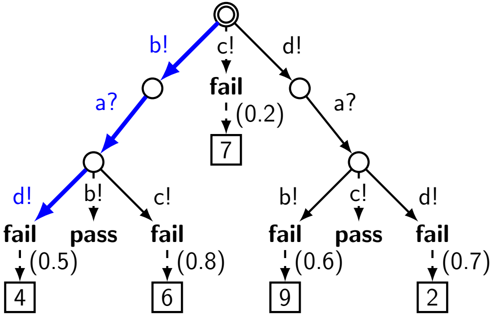

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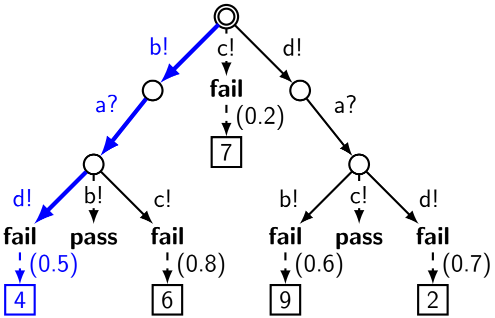
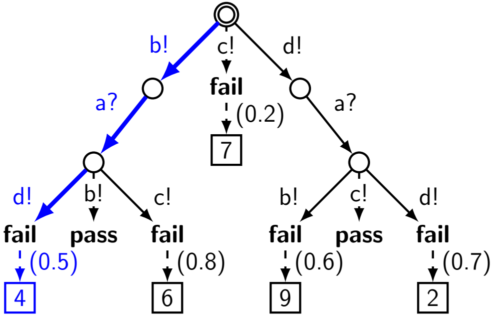


$$faultCov(b!\ a?\ d!) \quad =$$

Actual coverage

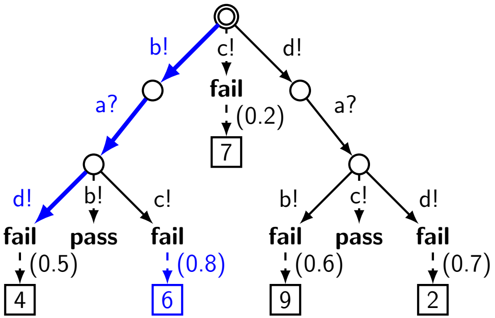
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$faultCov(b!\ a?\ d!) \quad = \quad 4$$

Actual coverage

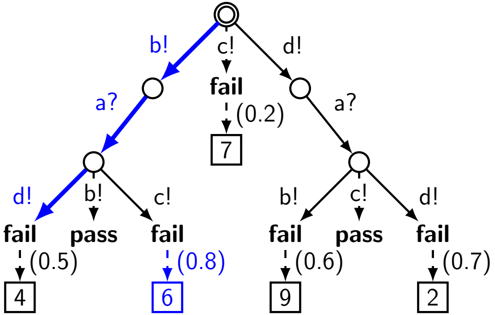
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$\begin{aligned} faultCov(b!\ a?\ d!) &= 4 \\ faultCov(b!\ a?\ c!) &= \end{aligned}$$

Actual coverage

Actual coverage of an execution or sequence of executions:
The sum of all fault coverages

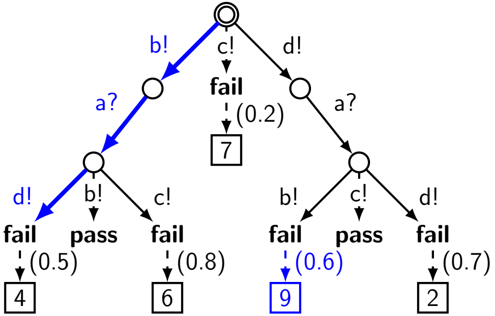


$$faultCov(b!\ a?\ d!) = 4$$
$$faultCov(b!\ a?\ c!) = 4.8$$

$$(1 - (1 - 0.8)) \cdot 6 = 4.8$$

Actual coverage

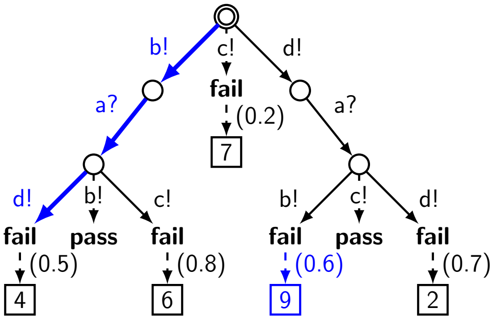
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$\begin{aligned} faultCov(b!\ a?\ d!) &= 4 \\ faultCov(b!\ a?\ c!) &= 4.8 \\ faultCov(d!\ a?\ b!) &= \end{aligned}$$

Actual coverage

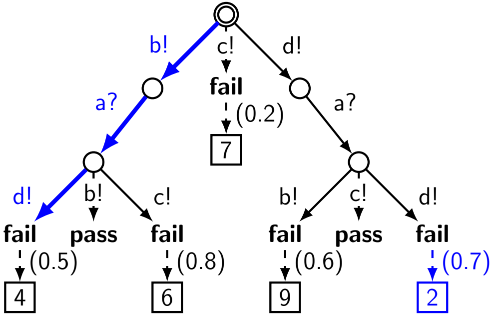
Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



$$
\begin{aligned}
faultCov(b!\ a?\ d!) &= 4 \\
faultCov(b!\ a?\ c!) &= 4.8 \\
faultCov(d!\ a?\ b!) &= 0
\end{aligned}
$$

Actual coverage

Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



faultCov(b! a? d!)	=	4
faultCov(b! a? c!)	=	4.8
faultCov(d! a? b!)	=	0
faultCov(d! a? d!)	=	

Actual coverage

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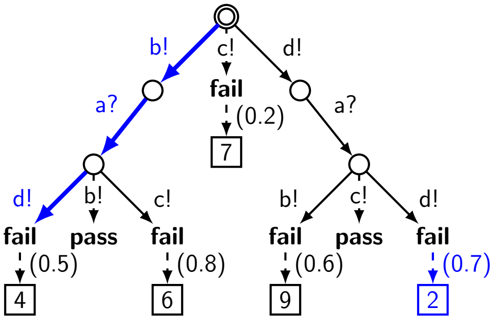

$$
\begin{aligned}
faultCov(b!\ a?\ d!) &= 4\\
faultCov(b!\ a?\ c!) &= 4.8\\
faultCov(d!\ a?\ b!) &= 0\\
faultCov(d!\ a?\ d!) &= 0
\end{aligned}
$$

Actual coverage

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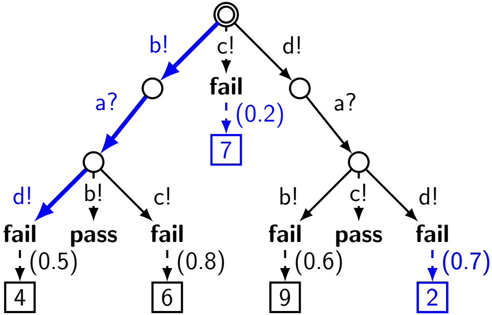

$$
\begin{aligned}
faultCov(b!\ a?\ d!) &= 4 \\
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faultCov(d!\ a?\ b!) &= 0 \\
faultCov(d!\ a?\ d!) &= 0 \\
faultCov(c!) &=
\end{aligned}
$$

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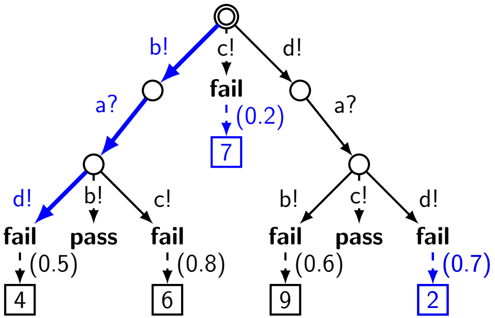
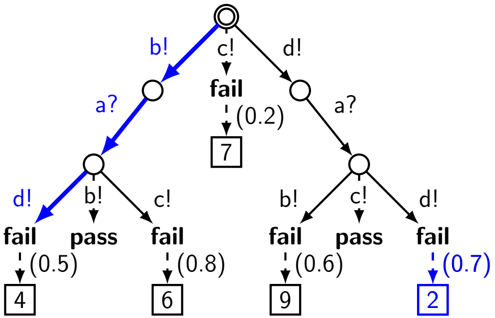


$$
\begin{aligned}
faultCov(b!\ a?\ d!) &= 4\\
faultCov(b!\ a?\ c!) &= 4.8\\
faultCov(d!\ a?\ b!) &= 0\\
faultCov(d!\ a?\ d!) &= 0\\
faultCov(c!) &= 1.4
\end{aligned}
$$

$$\boxed{(1-(1-0.2))\cdot 7 = 1.4}$$

Actual coverage

Actual coverage of an execution or sequence of executions:
The sum of all fault coverages



faultCov(b! a? d!)	=	4
faultCov(b! a? c!)	=	4.8
faultCov(d! a? b!)	=	0
faultCov(d! a? d!)	=	0
faultCov(c!)	=	1.4
absCov	=	10.2

Contents

Actual coverage of test cases

Actual coverage distribution of a test case

The *actual coverage distribution* of a test case predicts its actual coverage.

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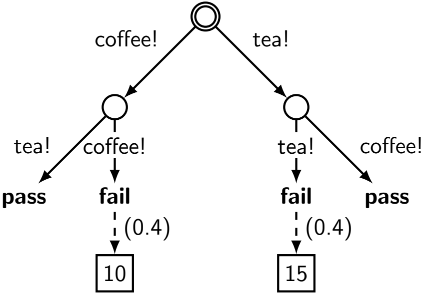

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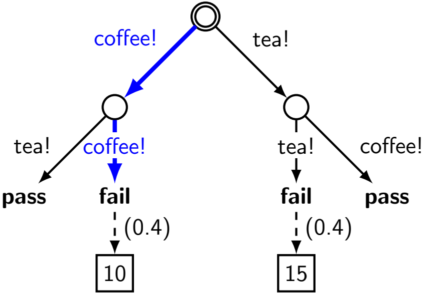

absCov
10

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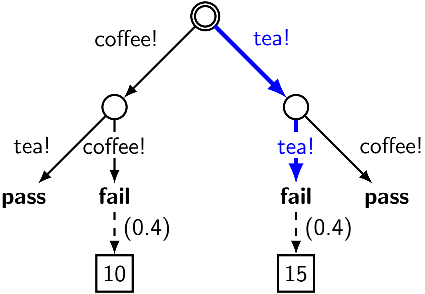

absCov
10
15

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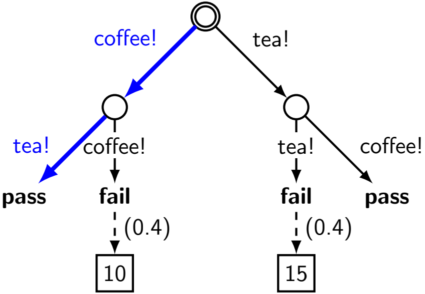

absCov
10
15
4

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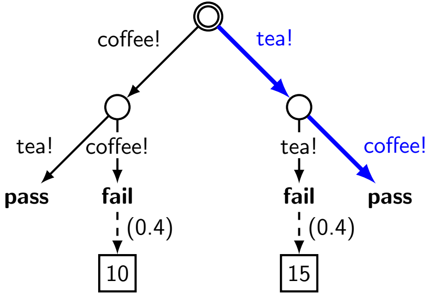

absCov
10
15
4
6

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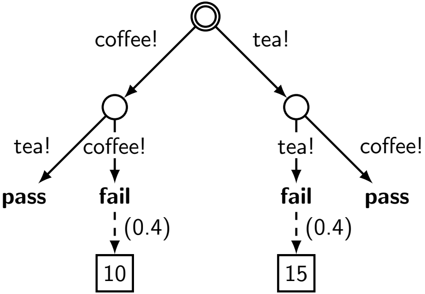

absCov	$\mathbb{P}$
10	0.015
15	0.005
4	0.735
6	0.245

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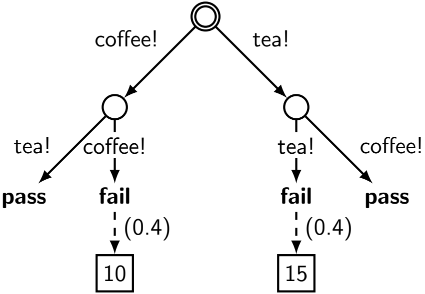

absCov	$\mathbb{P}$	$\times$
10	0.015	0.150
15	0.005	0.075
4	0.735	2.940
6	0.245	1.470

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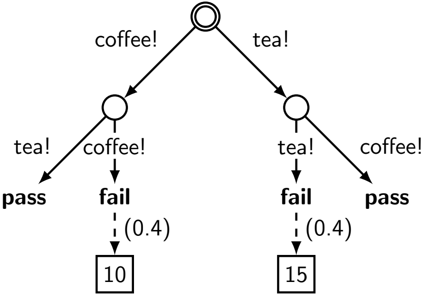
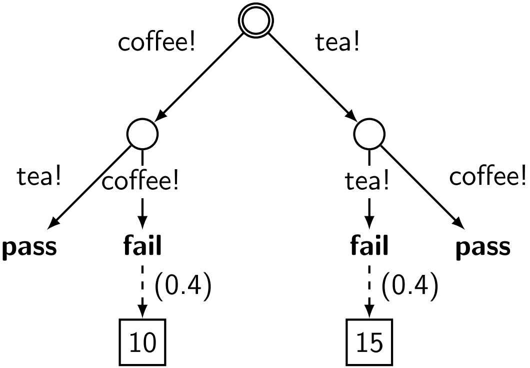


absCov	$\mathbb{P}$	$\times$
10	0.015	0.150
15	0.005	0.075
4	0.735	2.940
6	0.245	1.470 +
		$\mathbb{E}(absCov) = 4.635$

Branching probabilities

Branching probabilities

The *branching probabilities* p^{br} describe how the implementation is expected to behave



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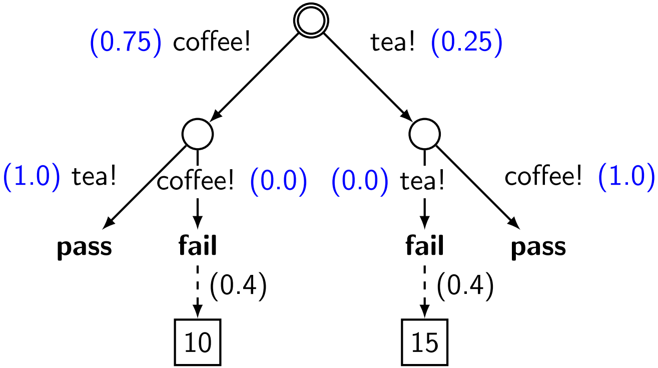

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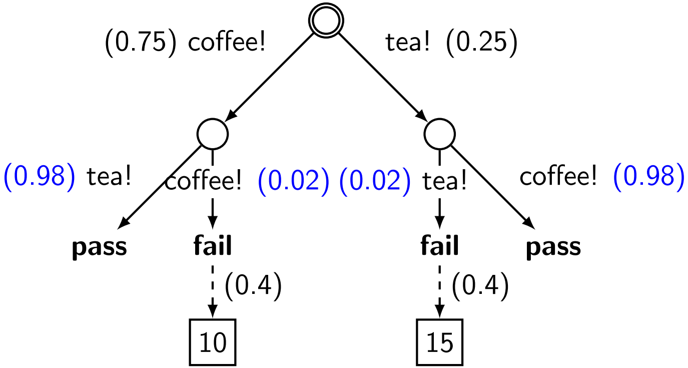
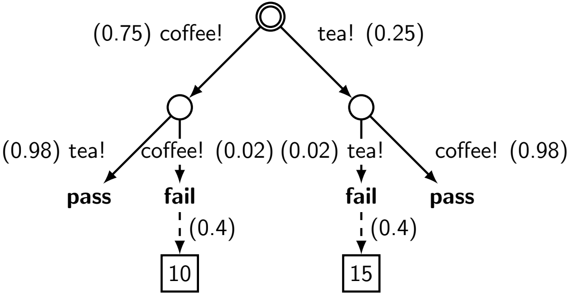
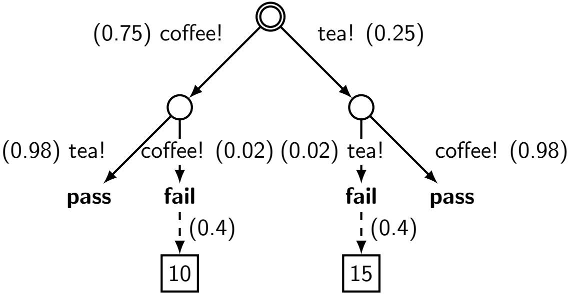


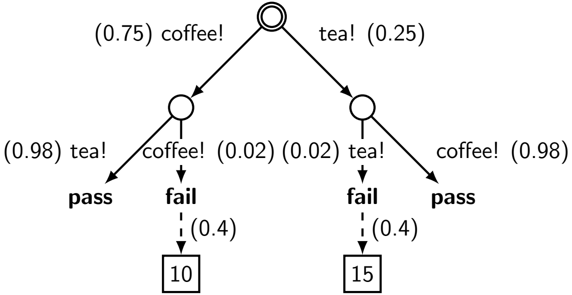
Trace occurrence probabilities and actual coverage



Trace occurrence probabilities and actual coverage



$$p^{to}(\mathit{coffee!}\ \ \mathit{tea!}) = 0.75 \cdot 0.98 = 0.735$$



$$p^{to}(\mathit{coffee!}\ \mathit{tea!}) = 0.75 \cdot 0.98 = 0.735$$

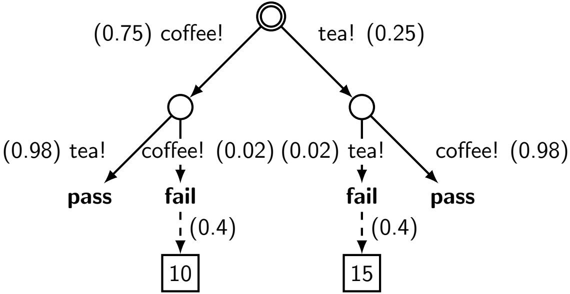
$$p^{to}(\mathit{coffee!}\ \mathit{coffee!}) = 0.75 \cdot 0.02 = 0.015$$

$$p^{to}(\mathit{tea!}\ \mathit{tea!}) = 0.75 \cdot 0.02 = 0.005$$

$$p^{to}(\mathit{tea!}\ \mathit{coffee!}) = 0.25 \cdot 0.98 = 0.245$$

Sequences of executions

- Suppose we perform three executions of



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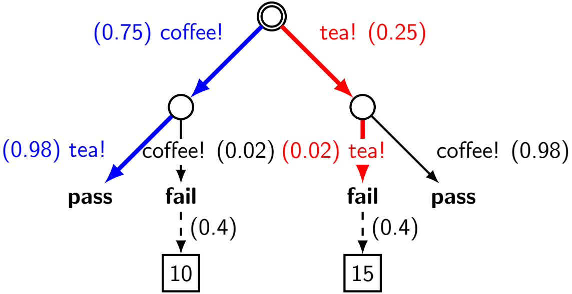
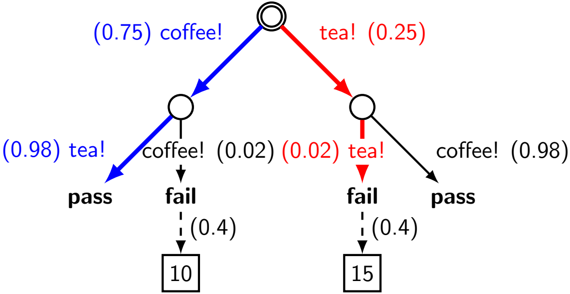


- Possible observation: [blue, blue, red]

Sequences of executions

- Suppose we perform three executions of

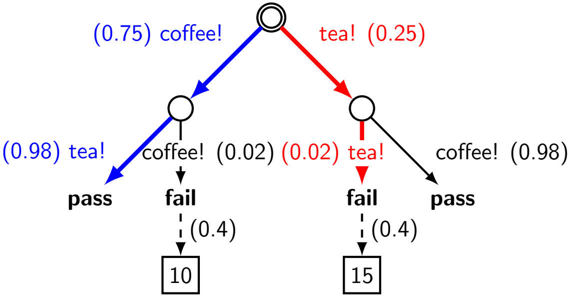


- Possible observation: [blue, blue, red]
- Actual coverage:

$$15 +$$

Sequences of executions

- Suppose we perform three executions of

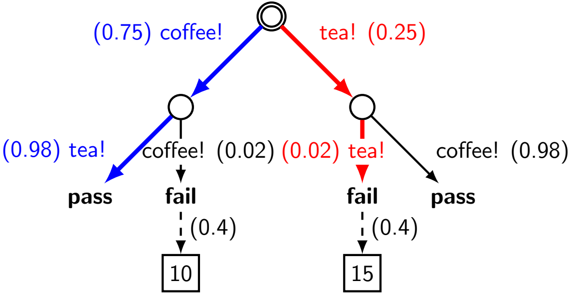


- Possible observation: [blue, blue, red]
- Actual coverage:

$$15 + (1 - (1 - 0.4)^2) \cdot 10$$

Sequences of executions

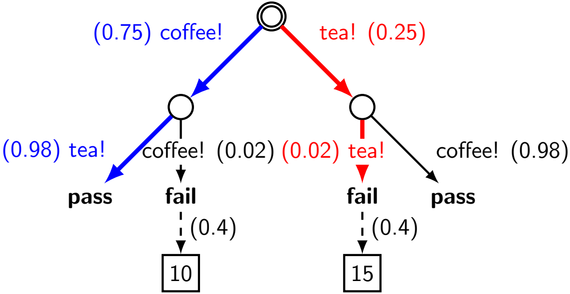
- Suppose we perform three executions of



- Possible observation: [blue, blue, red]
- Actual coverage:

$$15 + (1 - (1 - 0.4)^2) \cdot 10 = 15 + 6.4 = 21.4$$

- Suppose we perform three executions of



- Possible observation: [blue, blue, red]
- Actual coverage:

$$15 + (1 - (1 - 0.4)^2) \cdot 10 = 15 + 6.4 = 21.4$$

- Many observations possible: $O(|exec|^n)$

Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

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$\boxed{\mathbb{E}(p^{cov}(\sigma a))}$

Expected actual coverage for a sequence of executions

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Case 1: presence shown

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

Case 1: presence shown
$\mathbb{P}$[observe σa]

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

Case 1: presence shown
$\mathbb{P}$[not observe σa]

Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - {\color{blue}(1 - p^{to}(\sigma a))^n}) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

Case 1: presence shown
$\mathbb{P}$[never observe σa]

Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

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$\mathbb{P}$[ever observe σa]

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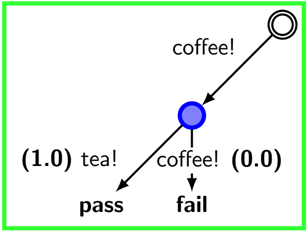
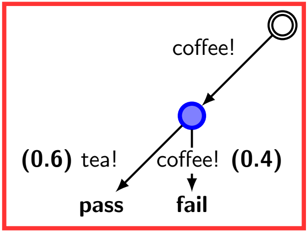
$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

Case 2: presence not shown
Absence shown $0 \ldots n$ times

Coverage probabilities





- Conditional branching probability: 0.4 ($\mathbb{P}$[*coffee! produced from blue* | *coffee! possible from blue*])
- Observation: 5 times *coffee! tea!*
- Probability of not even one *coffee!*:

$$(1 - p^{cbr})^5 = 0.6^5 = 0.08$$

- Probability of at least one *coffee!*:

$$1 - (1 - p^{cbr})^5 = 1 - 0.6^5 = 0.92$$

- Coverage probability:

$$\boxed{p^{cov} = 1 - (1 - p^{cbr})^k}$$

Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

Case 2: presence not shown
Absence shown $0 \ldots n$ times

Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

Case 2: presence not shown
$\mathbb{P}$[exactly k times σ]

Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

Case 2: presence not shown
$\mathbb{P}[k \text{ times } \sigma]$

Expected actual coverage for a sequence of executions

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Case 2: presence not shown
$\mathbb{P}$[the others not σ]

Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

Case 2: presence not shown
All possible orderings

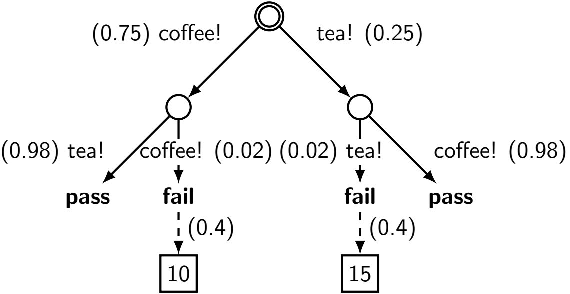
Expected actual coverage for a sequence of executions

Theorem

$$\mathbb{E}(actCov_n) = \sum_{\sigma a \in err_t} f(\sigma a) \cdot \Bigg((1 - (1 - p^{to}(\sigma a))^n) \cdot 1 + \sum_{k=0}^{n} \binom{n}{k} p^{to}(\sigma)^k (1 - p^{to}(\sigma))^{n-k} \cdot (1 - p^{br}(a \mid \sigma))^k \cdot (1 - (1 - p^{cbr}(a \mid \sigma)^k)) \Bigg)$$

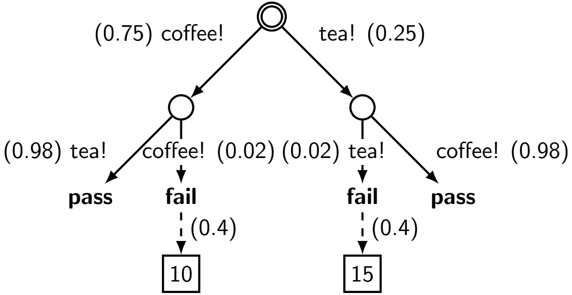
Case 2: presence not shown
$\mathbb{P}$[no presence shown]

Example of expected actual coverage



$$
\begin{aligned}
&10 \cdot \Big(\; (1-(1-0.75\cdot 0.02)^5) + \\
&\quad \sum_{k=0}^{5} \binom{5}{k} 0.75^k \cdot (1-0.75)^{5-k} \cdot (1-0.02)^k \cdot (1-(1-0.4)^k) \Big) + \\
&15 \cdot \Big(\; (1-(1-0.25\cdot 0.02)^5) + \\
&\quad \sum_{k=0}^{5} \binom{5}{k} 0.25^k \cdot (1-0.25)^{5-k} \cdot (1-0.02)^k \cdot (1-(1-0.4)^k) \Big) \\
&= 14.7
\end{aligned}
$$

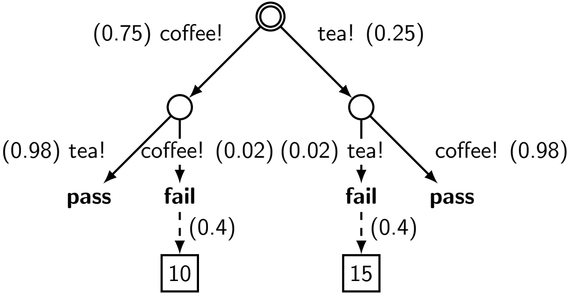
Example of expected actual coverage



$$\mathbb{E}(actCov_1) = 4.6$$

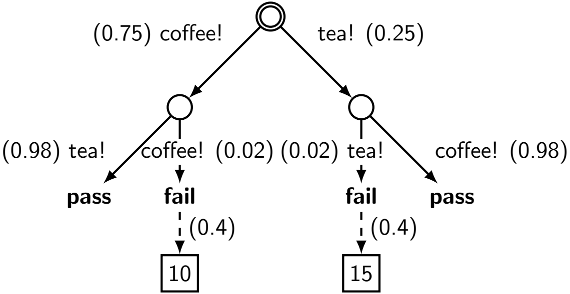
$$\mathbb{E}(actCov_5) = 14.7$$

Example of expected actual coverage



$$
\begin{aligned}
\mathbb{E}(actCov_1) &= 4.6 \\
\mathbb{E}(actCov_2) &= 8.2 \\
\mathbb{E}(actCov_3) &= 10.9 \\
\mathbb{E}(actCov_4) &= 13.0 \\
\mathbb{E}(actCov_5) &= 14.7
\end{aligned}
$$

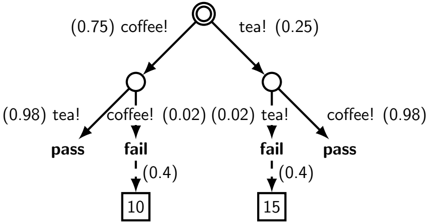
Example of expected actual coverage

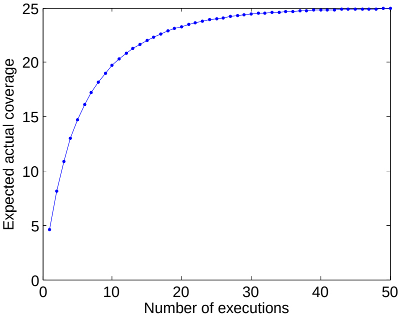


$$\mathbb{E}(actCov_1) = 4.6$$
$$\mathbb{E}(actCov_2) = 8.2$$
$$\mathbb{E}(actCov_3) = 10.9$$
$$\mathbb{E}(actCov_4) = 13.0$$
$$\mathbb{E}(actCov_5) = 14.7$$

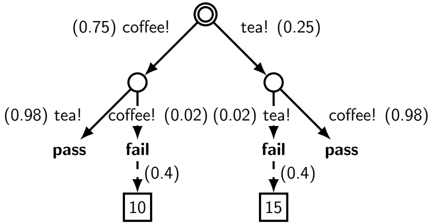
$$\mathbb{E}(actCov_{10}) = 19.7$$
$$\mathbb{E}(actCov_{25}) = 24.0$$
$$\mathbb{E}(actCov_{50}) = 24.9$$

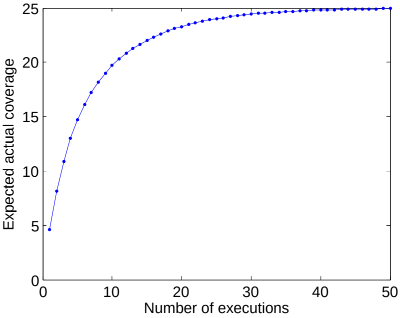
Asympotical behaviour of actual coverage





Asympotical behaviour of actual coverage





Theorem

$$\lim_{n\to\infty} \mathbb{E}(absCov_n) = absPotCov$$

Contents

Actual coverage for test suites

- Very similar to actual coverage for test cases: sum all the fault coverages
- Take into account in how many test cases an erroneous trace is contained
- Again, an efficient formula for the expected actual coverage exists

Theorem

$$\lim_{n \to \infty} \mathbb{E}(absCov_n) = absPotCov$$

Contents

Conclusions

Main results – what you saw today

- New notion of coverage: *actual coverage*
- Evaluating actual coverage of a given execution
- Predicting actual coverage of a test case or test suite

Conclusions

Main results – what you saw today

- New notion of coverage: *actual coverage*
- Evaluating actual coverage of a given execution
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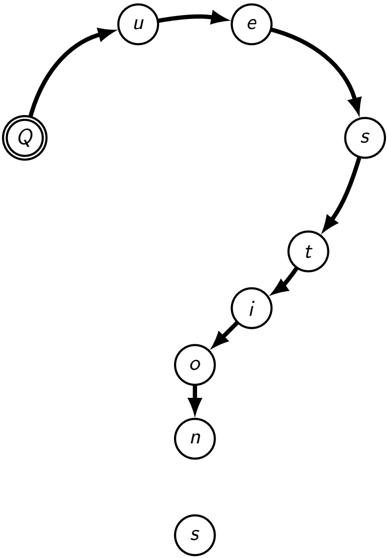
Main results – what I did not show

- Probabilistic fault automata
- Methods to derive p^{cbr} and p^{br}
- Mathematical proofs
- Detailed example
- Extra features:
    - Risk-based testing
    - Alternative approach to coverage probabilities
    - Approximations

Future work

Directions for future work

- Validation of the framework: tool support, case studies
- Dependencies between errors
- Accuracy of approximations
- On-the-fly test derivation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