UNIVERSITY OF TWENTE.

Formal Methods & Tools.

Efficient Modelling and Generation of Markov Automata

Mark Timmer

March 20, 2012

FMT Lunchmeeting

Joint work with Joost-Pieter Katoen, Jaco van de Pol, and Mariëlle Stoelinga

The overall goal: efficient and expressive modelling

Specifying systems with

- Nondeterminism
- Probability
- Timing

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(Nondeterminism and Probability) ← Probabilistic Automata (PAs)

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Interactive Markov Chains (IMCs)

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← Markov Automata (MAs)

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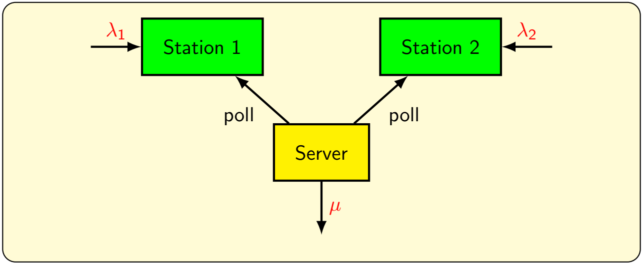

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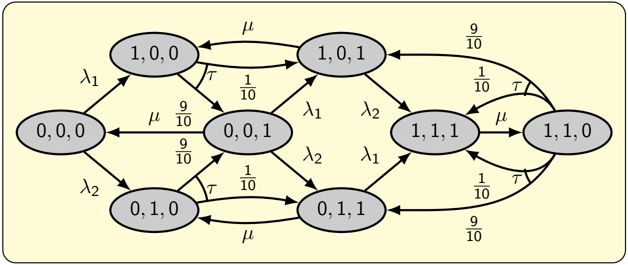

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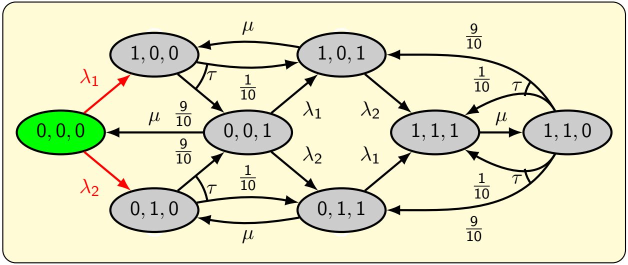

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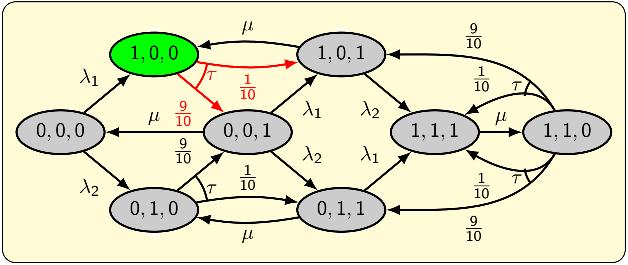

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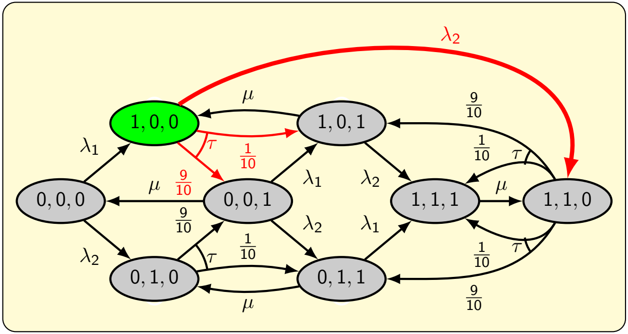

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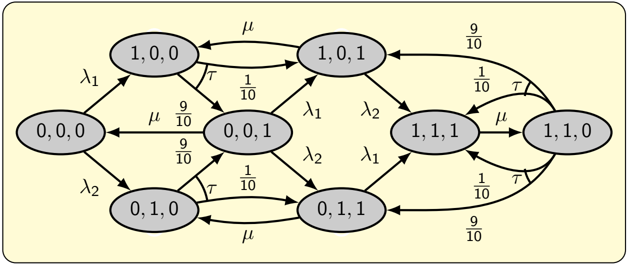

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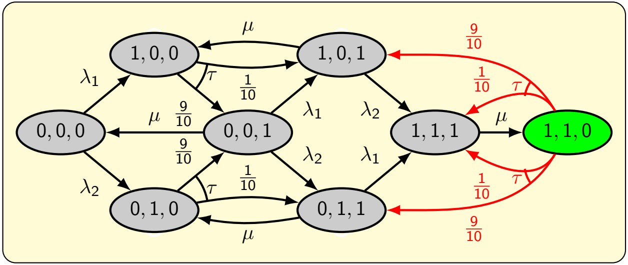

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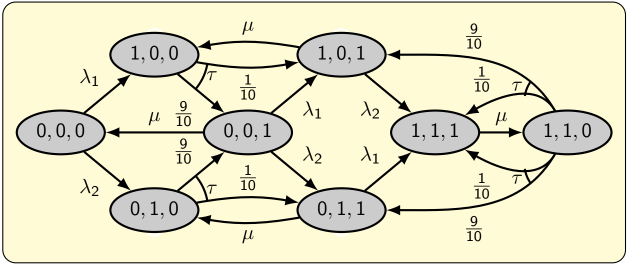
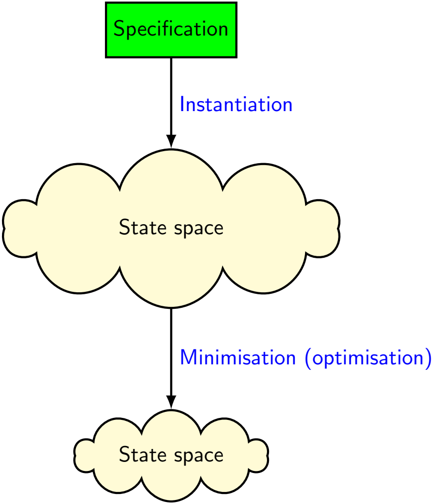


Observed limitations:

- No easy process-algebraic modelling language with data
- Susceptible to the state space explosion problem

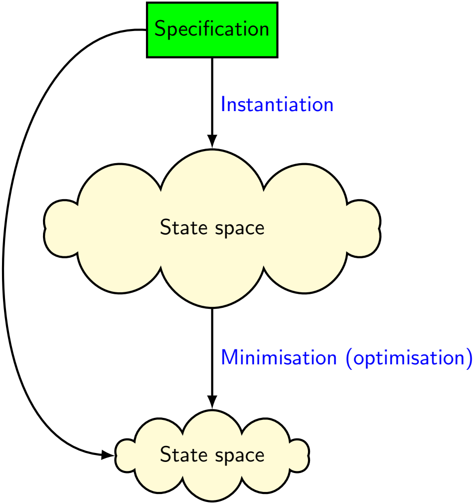
Combating the state space explosion



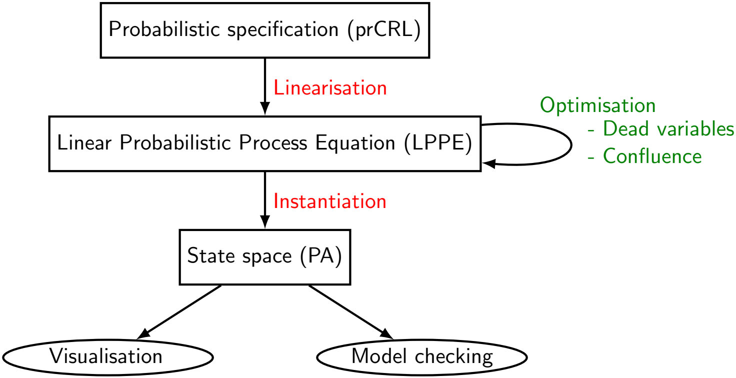
Combating the state space explosion

Optimised instantiation

- Dead variable reduction
- Confluence reduction



Earlier approach in the PA context



Current approach: extending and reusing

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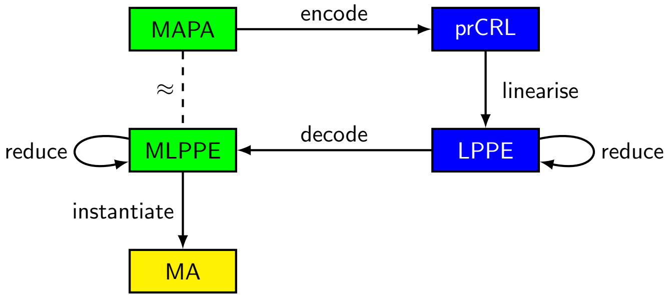

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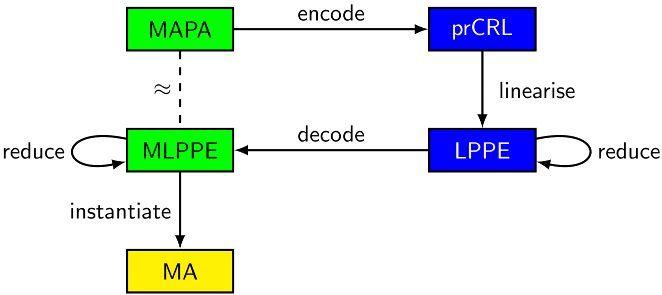
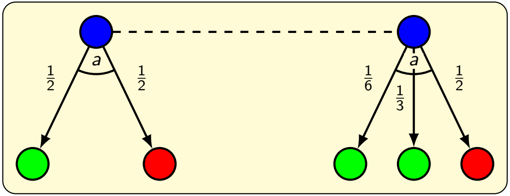


- Bisimulation-preserving transformations on prCRL do not necessarily preserve bisimulation on MAPA!

Strong bisimulation for Markov automata



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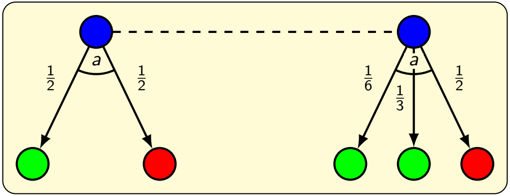

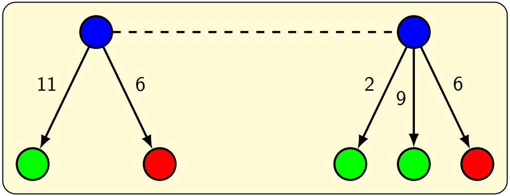

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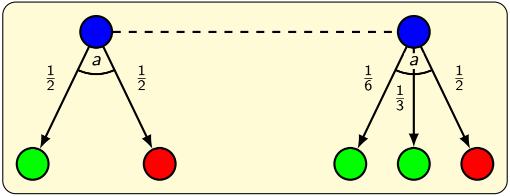

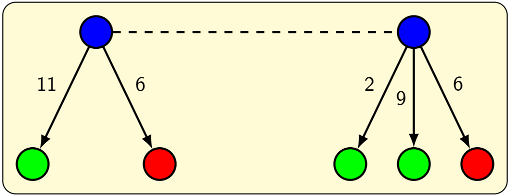

(If a state enables a τ-transition, all rates are ignored.)

Contents

A process algebra with data for MAs: MAPA

Specification language MAPA:

- Based on prCRL: data and probabilistic choice
- Additional feature: Markovian rates
- Semantics defined in terms of Markov automata
- Minimal set of operators to facilitate formal manipulation
- Syntactic sugar easily definable

A process algebra with data for MAs: MAPA

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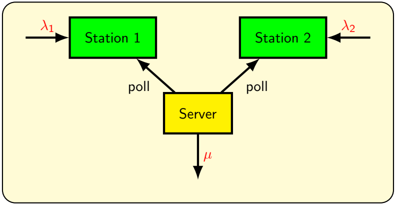
The grammar of MAPA

Process terms in MAPA are obtained by the following grammar:

$$p ::= Y(\boldsymbol{t}) \mid c \Rightarrow p \mid p + p \mid \sum_{x:D} p \mid a(\boldsymbol{t}) \sum\!\!\!\bullet_{x:D} f : p \mid (\lambda(\boldsymbol{t})) \cdot p$$

An example specification



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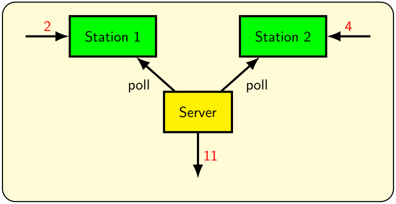

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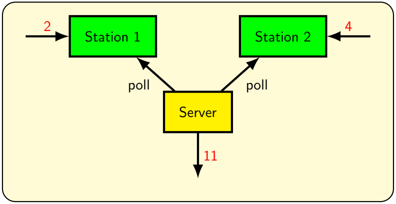

- There are 10 types of jobs
- The type of job that arrives is chosen nondeterministically
- Service time depends on job type (hence, we need queues)

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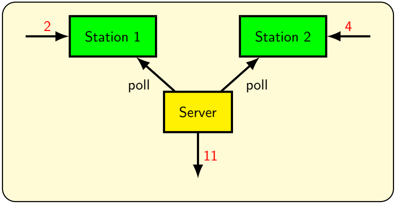

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The specification of the stations:

type *Jobs* $= \{1, \ldots, 10\}$

$\mathit{Station}(i : \{1,2\}, q : \text{Queue})$

$\quad = \mathbf{notFull}(q) \quad \Rightarrow (2i) \,.\, \sum_{j:\mathit{Jobs}} \mathit{arrive}(j).\mathit{Station}(i, \mathbf{enqueue}(q, j))$

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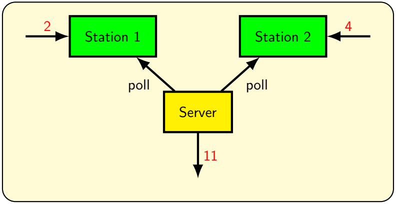

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$$\begin{aligned}
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&\quad = \textbf{notFull}(q) \quad\ \Rightarrow (2i) \,.\, \textstyle\sum_{j:\mathit{Jobs}} \mathit{arrive}(j).\mathit{Station}(i, \textbf{enqueue}(q, j)) \\
&\quad + \textbf{notEmpty}(q) \Rightarrow \mathit{deliver}(i, \textbf{head}(q)) \sum_{i \in \{1,9\}} \tfrac{i}{10} : i = 1 \Rightarrow \mathit{Station}(i, q) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + i = 9 \Rightarrow \mathit{Station}(i, \textbf{tail}(q))
\end{aligned}$$

Derivation-based operational semantics

$$\textsc{ActionPrefix}\ \frac{-}{a \cdot p \xrightarrow{a} p}$$

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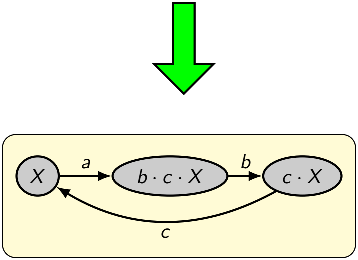

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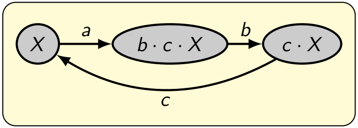

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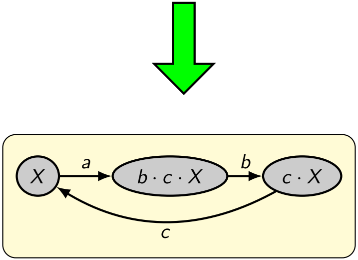

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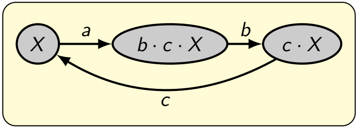

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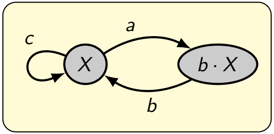

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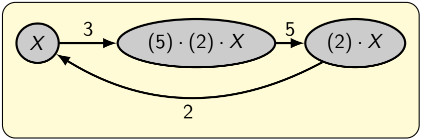

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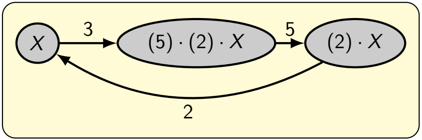

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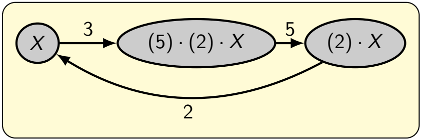

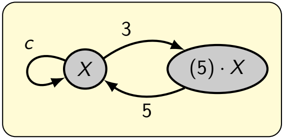

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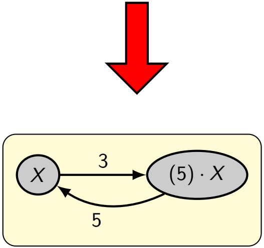

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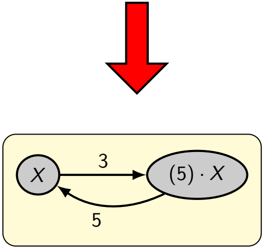

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As a solution, we look at derivations:

Derivation-based operational semantics

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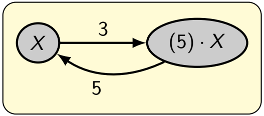

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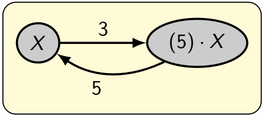

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$$X \xrightarrow{3}_{\langle SL,MP \rangle} (5) \cdot X$$

$$X \xrightarrow{3}_{\langle SR,MP \rangle} (5) \cdot X$$

Hence, the total rate from X to $(5) \cdot X$ is $3 + 3 = 6$.

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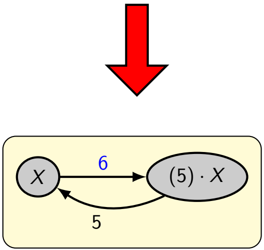

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MLPPEs

We defined a special format for MAPA, the MLPPE:

$$X(\boldsymbol{g} : \boldsymbol{G}) = \sum_{i \in I} \sum_{\boldsymbol{d_i} : \boldsymbol{D_i}} c_i \Rightarrow a_i(\boldsymbol{b_i}) \sum\nolimits_{\bullet\, \boldsymbol{e_i} : \boldsymbol{E_i}} f_i : X(\boldsymbol{n_i})$$

$$+ \sum_{j \in J} \sum_{\boldsymbol{d_j} : \boldsymbol{D_j}} c_j \Rightarrow (\lambda_j(\boldsymbol{b_j})) \cdot X(\boldsymbol{n_j})$$

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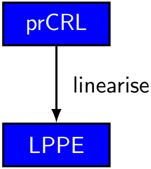
Advantages of using MLPPEs instead of MAPA specifications:

- Easy state space generation
- Straight-forward parallel composition
- **Symbolic optimisations enabled at the language level**

Encoding into prCRL



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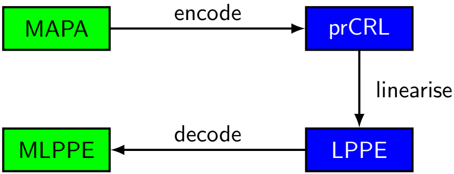

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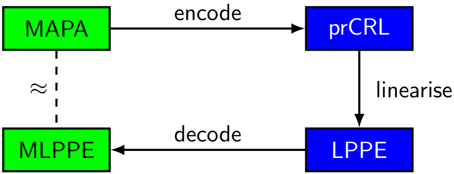

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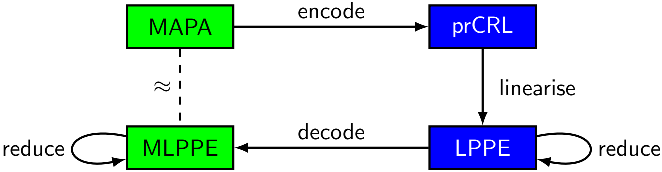

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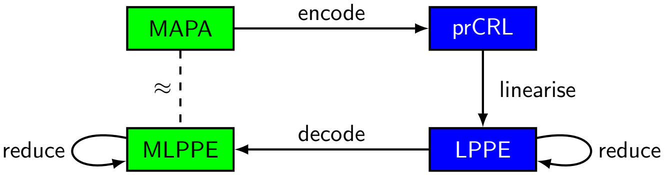

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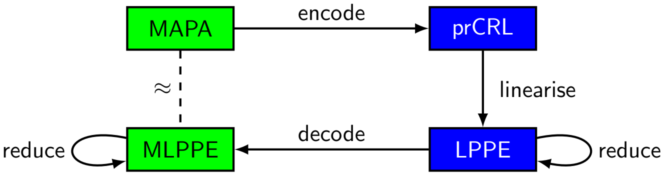

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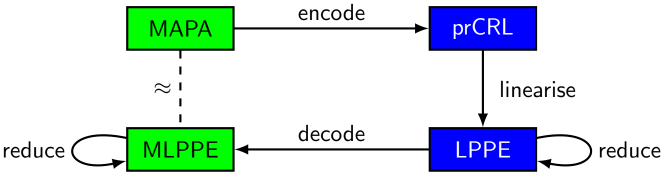

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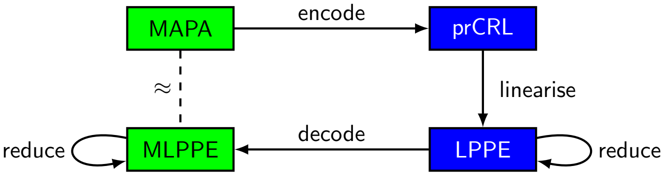

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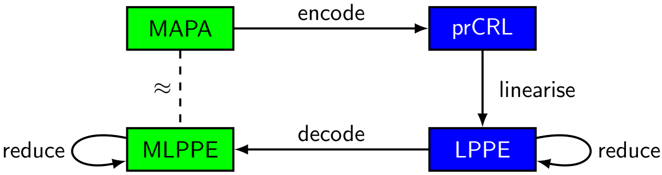

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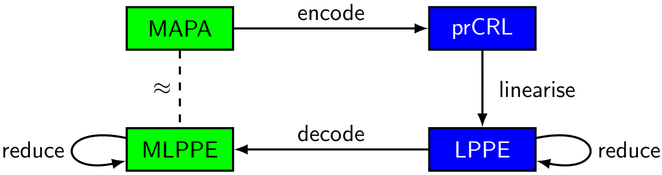

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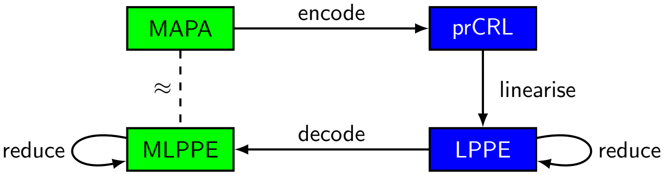

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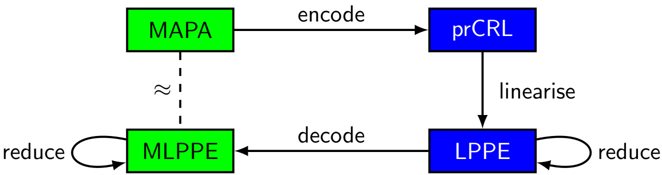

Possible solution: encode a rate λ as action rate$_i(\lambda)$.

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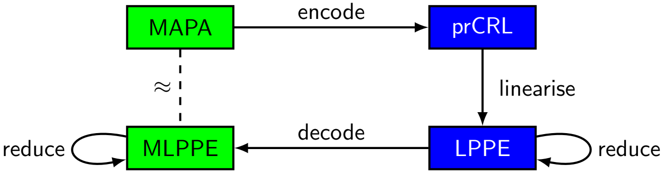

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Problem:

Even isomorphic prCRL specifications might yield different MLPPEs.

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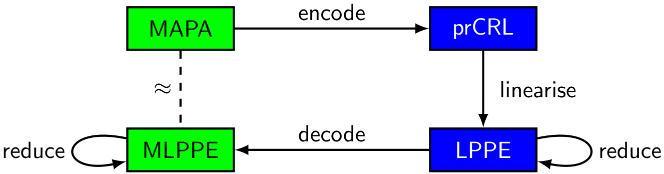

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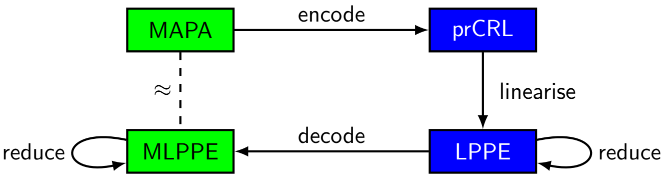

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Stronger equivalence on prCRL specifications needed!

Derivation-preserving bisimulation

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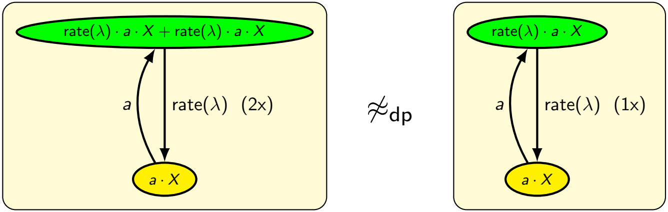

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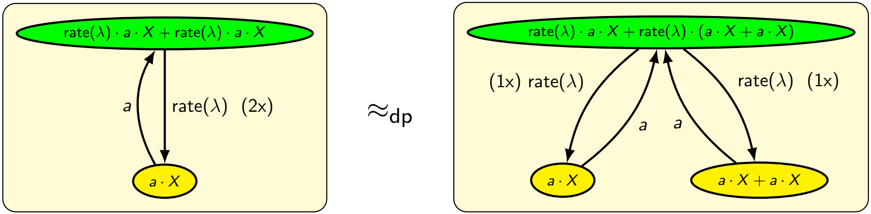

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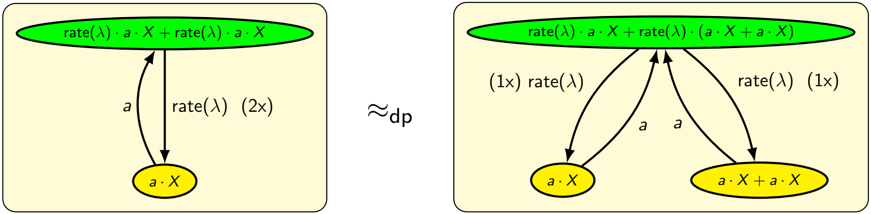

Proposition

Derivation-preserving bisimulation is a congruence for prCRL.

Derivation-preserving bisimulation: important results

Theorem

Given a derivation-preserving prCRL transformation f,

$$\text{decode}(f(\text{encode}(M))) \approx M$$

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Corollary

The linearisation procedure of prCRL can be reused for MAPA.

Generalising existing reduction techniques

Existing reduction techniques that preserve derivations:

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- Dead variable reduction

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$X(id : Id) = print(id) \cdot X(id)$

init $X(Mark)$

$\rightarrow$

$X = print(Mark) \cdot X$

init X

Generalising existing reduction techniques

Existing reduction techniques that preserve derivations:

- Constant elimination
- Expression simplification
- Dead variable reduction

$X = (3 = 1 + 2 \vee x > 5) \Rightarrow beep \cdot Y$

$\rightarrow$

$X = beep \cdot Y$

Generalising existing reduction techniques

Existing reduction techniques that preserve derivations:

- Constant elimination
- Expression simplification
- Dead variable reduction

- Deduce the control flow of an (M)LPPE
- Examine relevance (liveness) of variables
- Reset dead variables

Generalising existing reduction techniques

Implementation of dead variable reduction for prCRL:

```haskell
                                                  ]
                              | otherwise = []

usedInRelevantNextState :: LPPE -> Int -> Int -> Relevance -> Int -> String -> Bool
usedInRelevantNextState lppe summandNr dk relevant cfp target = or [variableInExpression parameter (getNextState summand l) | (l,cfpp,targett) <- relevant, cfp == cfpp, target == targett]
  where
    summand   = getPSummand lppe summandNr
    parameter = fst ((getLPPEPars lppe)!!dk)

indirectRelevance3 :: DataSpec -> LPPE -> [Int] -> Rules -> BelongsTo -> Relevance -> [Int] -> Relevance
indirectRelevance3 dataspec lppe cfps rules belongsTo relevant [dk, dj] | elem (dk, dj) belongsTo = [(dk, dj, value) | i <- getPSummandNrs lppe,
                                                  indirectRelevance3_ dataspec lppe cfps i dk dj rules belongsTo relevant,
                                                  value <- [sourceValue dataspec lppe i dj]]
                              | otherwise = []

indirectRelevance3_ :: DataSpec -> LPPE -> [Int] -> Int -> Int -> Int -> Rules -> BelongsTo -> Relevance -> Bool
indirectRelevance3_ dataspec lppe cfps summandNr dk dj rules belongsTo relevant = valid
  where
    summand    = getPSummand lppe summandNr
    parameters = [0..length (getLPPEPars lppe) - 1]
    valid = or [usedInRelevantNextState2 lppe summandNr dk belongsTo relevant dp (destinationValue dataspec lppe summandNr dp) dj | dp <- cfps, elem (summandNr, dp) rules]

usedInRelevantNextState2 lppe summandNr dk belongsTo relevant cfp target dj = or [variableInExpression parameter (getNextState summand l) | (l,cfpp,targett) <- relevant, cfp == cfpp, target == targett, no
  where
    summand   = getPSummand lppe summandNr
    parameter = fst ((getLPPEPars lppe)!!dk)


transformLPPE :: DataSpec -> LPPE -> Rules -> BelongsTo -> InitialState -> [Int] -> [Int] -> Relevance -> LPPE
transformLPPE dataspec (LPPE name pars summands) rules belongsTo initial dps cfps relevant = LPPE name pars (transformSummands dataspec (LPPE name pars summands) rules belongsTo initial dps cfps relevant :

transformSummands dataspec lppe rules belongsTo initial dps cfps relevant [] i = []
transformSummands dataspec lppe rules belongsTo initial dps cfps relevant (s:ss) i = (transformSummand dataspec lppe rules belongsTo i initial dps cfps relevant s):
                                          (transformSummands dataspec lppe rules belongsTo initial dps cfps relevant ss (i+1))

transformSummand dataspec lppe rules belongsTo i initial dps cfps relevant (params, c, a, aps, probChoices, g) = newSummand
  where
    newG       = transformNextStates dataspec lppe rules belongsTo i initial dps cfps relevant g [0..length g - 1]
    newSummand = (params, c, a, aps, probChoices, newG)

transformNextStates dataspec lppe rules belongsTo i initial dps cfps relevant g []     = g
transformNextStates dataspec lppe rules belongsTo i initial dps cfps relevant g (k:ks) | reset && (g!!k) /= Variable (initial!!k) = transformNextStates2 dataspec lppe rules belongsTo i initial dps cfps re
                                          | otherwise = transformNextStates dataspec lppe rules belongsTo i initial dps cfps relevant g ks
  where
    newG = (take k g) ++ [Variable (initial!!k)] ++ (drop (k+1) g)
    relevanceInfo = [elem k dps && elem (i, cfp) rules && elem (k,cfp) belongsTo && not(elem (k, cfp, destinationValue dataspec lppe i cfp) relevant) | cfp <- cfps]
    reset = (relevanceInfo /= [] && or relevanceInfo)

transformNextStates2 dataspec lppe rules belongsTo i initial dps cfps relevant newG ks old new = transformNextStates dataspec lppe rules belongsTo i initial dps cfps relevant newG ks

improveInitialState :: DataSpec -> LPPE -> BelongsTo -> Relevance -> InitialState -> InitialState
improveInitialState dataspec lppe belongsTo relevant initial = [improveInitialValue dataspec lppe initial belongsTo relevant parNr (initial!!parNr) | parNr <- [0..length initial - 1]]

improveInitialValue :: DataSpec -> LPPE -> InitialState -> BelongsTo -> Relevance -> Int -> String -> String
improveInitialValue dataspec lppe initial belongsTo relevant parNr initValue | canChange && willChange = newValue
                                        | otherwise = initValue
  where
    canChange  = [c | (d,c) <- belongsTo, d == parNr, not(elem (d,c,initial!!c) relevant)] /= []
    willChange = shouldChange dataspec lppe parNr
    newValue   = getNewValue dataspec lppe parNr


shouldChange :: DataSpec -> LPPE -> Int -> Bool
shouldChange dataspec lppe i = [s | s <- getPSummands lppe, changedToConstant dataspec (getNextState s i)] /= []

changedToConstant dataspec (Variable v) = stringIsConstant dataspec v
changedToConstant dataspec _            = False

getNewValue :: DataSpec -> LPPE -> Int -> String
getNewValue dataspec lppe i = [getValue (getNextState s i) | s <- getPSummands lppe, changedToConstant dataspec (getNextState s i)]!!0

getValue (Variable v) = v
```

Generalising existing reduction techniques

Implementation of dead variable reduction for prCRL:

Implementation of dead variable reduction for MAPA:

$$\text{deadVarRed} = \text{decode} \circ \text{deadVarRedOld} \circ \text{encode}$$

Novel reduction techniques

New reduction techniques for MAPA:

- Maximal progress reduction
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$$X = \sum_{d:\{1,2,3\}} d = 2 \Rightarrow send(d) \cdot X$$
$$Y = \sum_{d:\{1,2,3\}} (5) \cdot Y$$

$\rightarrow$

$$X = send(2) \cdot X$$
$$Y = (15) \cdot Y$$

Novel reduction techniques

New reduction techniques for MAPA:

- Maximal progress reduction
- Summation elimination
- Transition merging

$$X = (5) \cdot \tau(\tfrac{1}{2} \rightarrow a \cdot X + \tfrac{1}{2} \rightarrow b \cdot X)$$

$\rightarrow$

$$\begin{aligned} X = &\ (2.5) \cdot a \cdot X \\ + &\ (2.5) \cdot b \cdot X \end{aligned}$$

Implementation and Case Study

Implementation in SCOOP:

- Programmed in Haskell
- Stand-alone and web-based interface
- Linearisation, optimisation, state space generation

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	Original				Reduced			
Specification	States	Trans.	MLPPE Size	Time	States	Trans.	MLPPE Size	Time
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pollingQueue-25-1	3,330	5,256	15 / 335	0.9	3,330	5,256	8 / 226	0.6
pollingQueue-100-1	50,805	81,006	15 / 335	15.9	50,805	81,006	8 / 226	11.7
pollingQueue-5-2	27,659	47,130	15 / 335	8.1	23,690	43,161	8 / 226	3.7
pollingQueue-5-2'	27,659	47,130	15 / 335	8.1	170	256	5 / 176	0.0
pollingQueue-7-2	454,667	778,266	15 / 335	136.4	389,642	713,241	8 / 226	60.2
pollingQueue-7-2'	454,667	778,266	15 / 335	136.2	306	468	5 / 176	0.0
pollingQueue-3-3	14,322	25,208	15 / 335	5.3	11,122	22,008	8 / 226	1.8
pollingQueue-3-4	79,307	143,490	15 / 335	36.1	57,632	121,815	8 / 226	9.9
pollingQueue-3-5	316,058	581,892	15 / 335	168.9	218,714	484,548	8 / 226	39.5
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Future Work:

- Generalise confluence reduction to MAs and MAPA
- Develop model checking techniques for MAs