# An extended test coverage framework

*From potential to actual coverage*

Mark Timmer

March 12, 2008

# Contents

# Introduction

## Motivation for research on testing

- Software is getting more and more complex
- Bugs cost a lot of money
- Testing is a large part of software development

# Introduction

## Motivation for research on testing

- Software is getting more and more complex
- Bugs cost a lot of money
- Testing is a large part of software development

## Motivation for research on test coverage

- Testing is inherently incomplete
- A notion of *quality* of a test suite is necessary

# Introduction

## Motivation for research on testing

- Software is getting more and more complex
- Bugs cost a lot of money
- Testing is a large part of software development

## Motivation for research on test coverage

- Testing is inherently incomplete
- A notion of *quality* of a test suite is necessary

## Motivation for my research project

- Previous work by Laura Brandán Briones, Marielle and Ed
- Ideas for several improvements

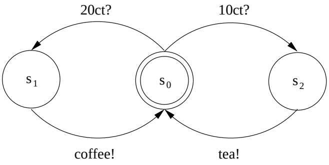
# Contents

# Preliminaries – labeled transition systems

## Definition LTSs

LTS $\mathcal{A} = \langle S, s^0, L, \Delta \rangle$, such that

- $S$: set of states
- $s^0$: initial state
- $L$: set of actions (partitioned into *input actions* and *output actions*)
- $\Delta$: transition relation (assumed deterministic)

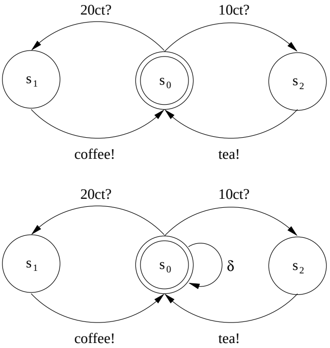


# Preliminaries – labeled transition systems

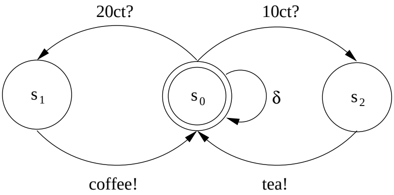
## Definition LTSs

LTS $\mathcal{A} = \langle S, s^0, L, \Delta \rangle$, such that

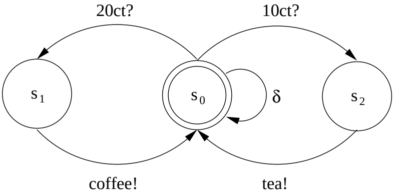
- $S$: set of states
- $s^0$: initial state
- $L$: set of actions (partitioned into *input actions* and *output actions*)
- $\Delta$: transition relation (assumed deterministic)

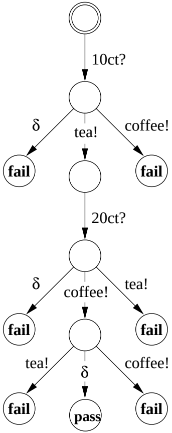


# Preliminaries – test cases for LTSs

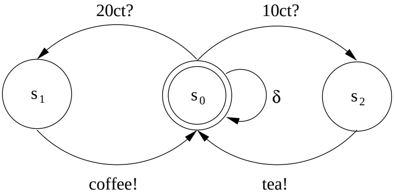


# Preliminaries – test cases for LTSs

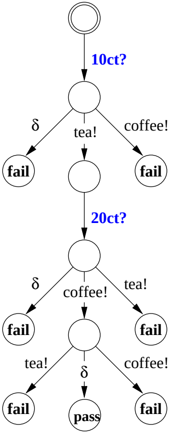


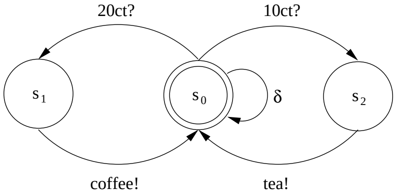
# Preliminaries – test cases for LTSs



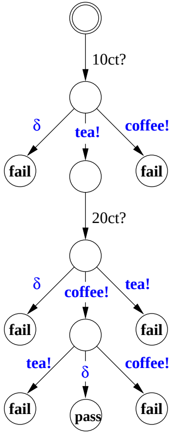

- Perform an input



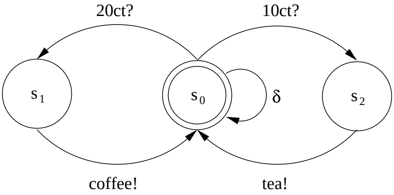
# Preliminaries – test cases for LTSs



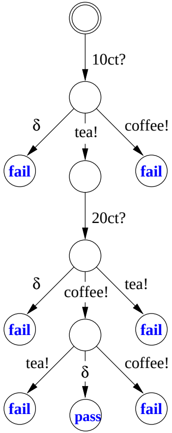

- Perform an input
- Observe all outputs



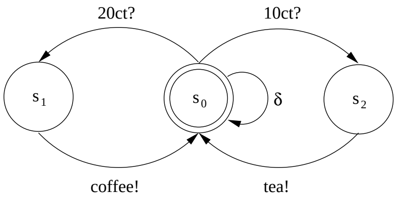
# Preliminaries – test cases for LTSs



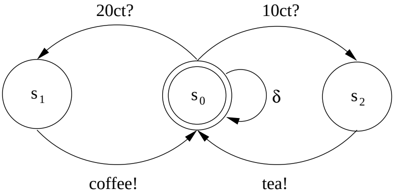

- Perform an input
- Observe all outputs
- Always stop after an error



# Preliminaries – Weighted fault models (WFMs)

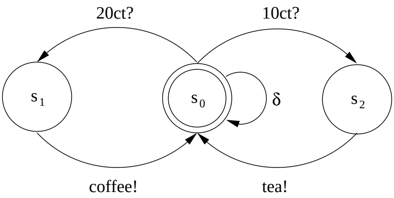


# Preliminaries – Weighted fault models (WFMs)



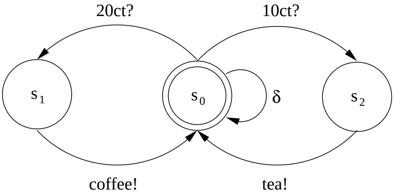

$$f(coffee!) \qquad = 10$$

# Preliminaries – Weighted fault models (WFMs)



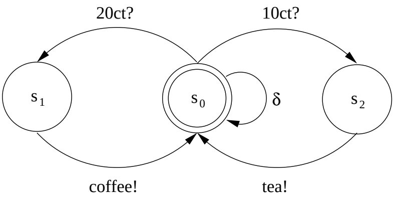

$$f(coffee!) = 10$$
$$f(10ct?\ tea!) = 0$$

# Preliminaries – Weighted fault models (WFMs)



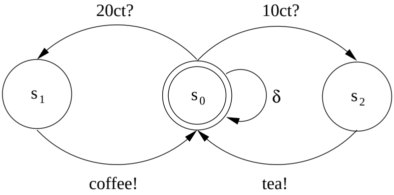

$$f(\textit{coffee!}) = 10$$
$$f(\textit{10ct? tea!}) = 0$$
$$f(\textit{10ct? coffee!}) = 5$$

# Preliminaries – Weighted fault models (WFMs)




$$
\begin{aligned}
f(\textit{coffee!}) &= 10\\
f(\textit{10ct? tea!}) &= 0\\
f(\textit{10ct? coffee!}) &= 5\\
f(\textit{10ct? tea! 10ct? coffee!}) &= 3
\end{aligned}
$$

# Preliminaries – Weighted fault models (WFMs)

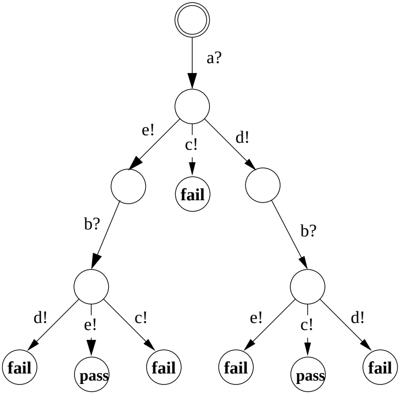



$$f(\mathit{coffee!}) = 10$$
$$f(\mathit{10ct?\ tea!}) = 0$$
$$f(\mathit{10ct?\ coffee!}) = 5$$
$$f(\mathit{10ct?\ tea!\ 10ct?\ coffee!}) = 3$$

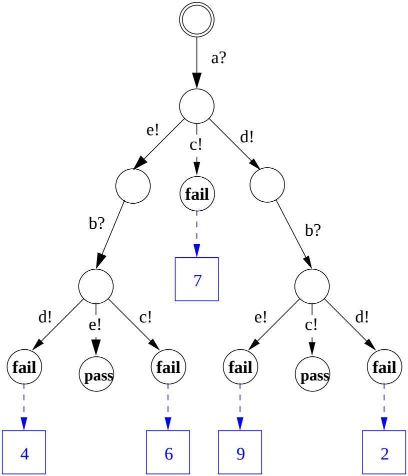
## Restriction on weighted fault models

$$0 < \sum_{\sigma \in L^*} f(\sigma) < \infty$$

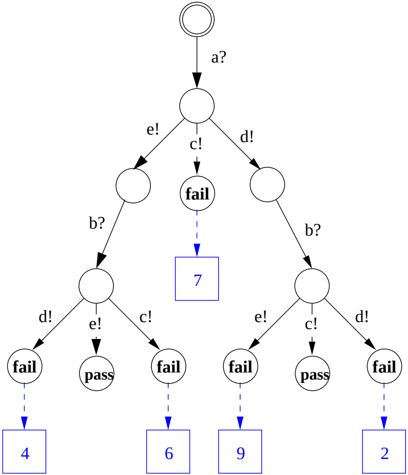
# Preliminaries – Weighted fault models (WFMs)



# Preliminaries – Weighted fault models (WFMs)

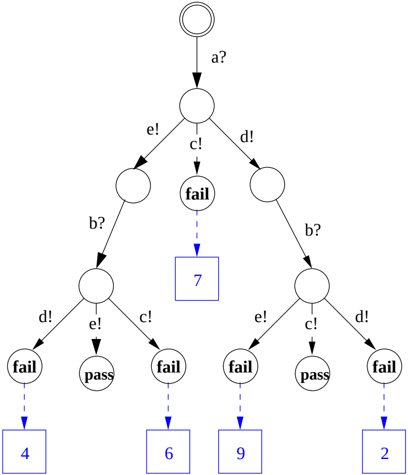


# Preliminaries – Weighted fault models (WFMs)




Assume $\sum_{\sigma \in L^*} f(\sigma) = 150$

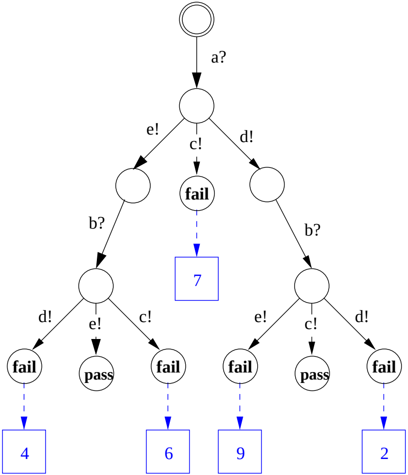
# Preliminaries – Weighted fault models (WFMs)




Assume $\sum_{\sigma \in L^*} f(\sigma) = 150$

$totCov_p = 150$

# Preliminaries – Weighted fault models (WFMs)

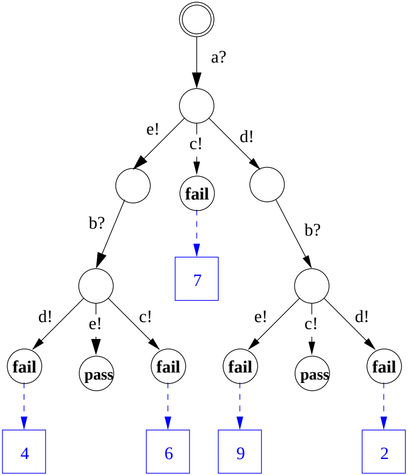



Assume $\sum_{\sigma \in L^*} f(\sigma) = 150$

$totCov_p = 150$

$absCov_p = 7 + 4 + 6 + 9 + 2 = 28$

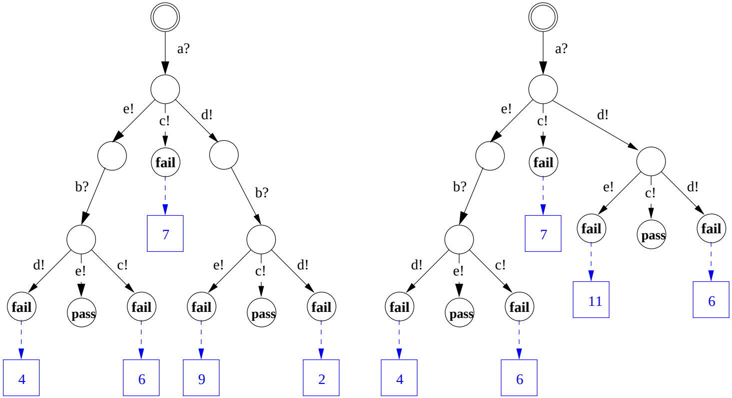
# Preliminaries – Weighted fault models (WFMs)



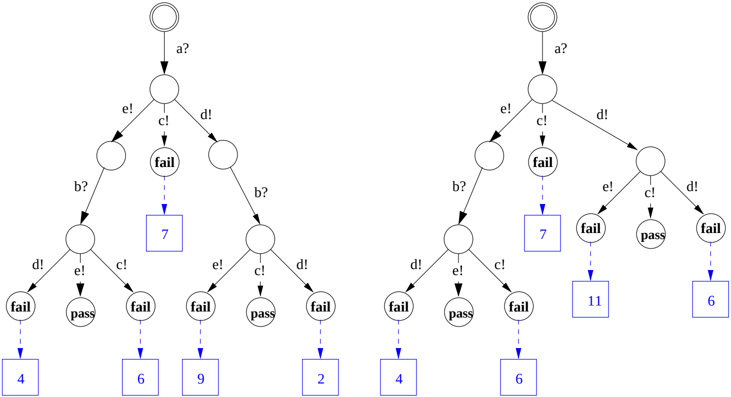

Assume $\sum_{\sigma \in L^*} f(\sigma) = 150$

$$totCov_p = 150$$
$$absCov_p = 7 + 4 + 6 + 9 + 2 = 28$$
$$relCov_p = \frac{28}{150} = 0.19$$

# Preliminaries – Weighted fault models (WFMs)

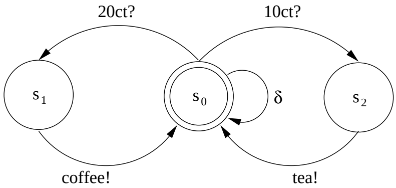


# Preliminaries – Weighted fault models (WFMs)

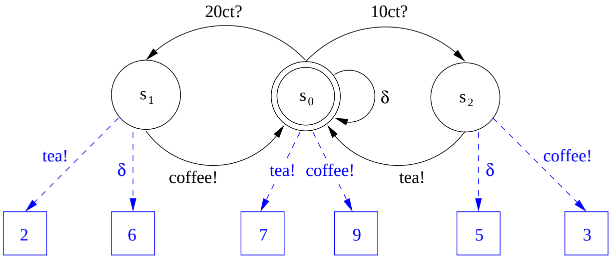



$$absCov_p = 7 + 4 + 6 + 9 + 2 + 11 + 6 = 45$$

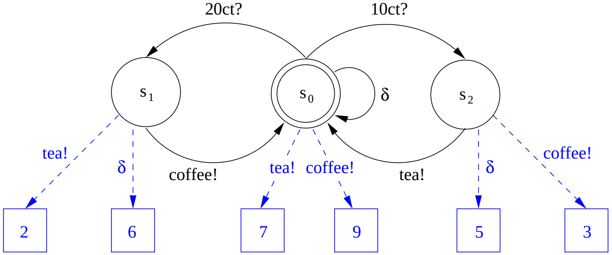
# Preliminaries - Fault automata



# Preliminaries - Fault automata



# Preliminaries - Fault automata




## Definition of fault automata (FAs)

Fault automaton: an LTS and a function $r$ assigning these weights. We require that $r(s, a!) = 0$ for correct outputs.

# From fault automaton to weighted fault model

Problem: infinite traces over FA, so $\sum_{\sigma \in L*} f(\sigma) \nless \infty$

# From fault automaton to weighted fault model

Problem: infinite traces over FA, so $\sum_{\sigma \in L^*} f(\sigma) \nless \infty$

Solutions:

- Discard traces with length larger than some threshold
- Discount error weights by their *depth*

# From fault automaton to weighted fault model

Problem: infinite traces over FA, so $\sum_{\sigma \in L^*} f(\sigma) \not< \infty$

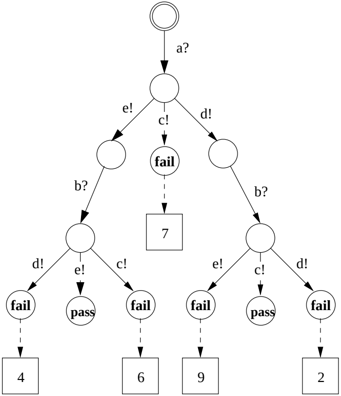
Solutions:

- Discard traces with length larger than some threshold
- Discount error weights by their *depth*

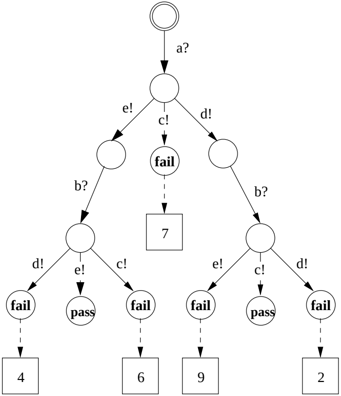
Not relevant for my work.

# Contents

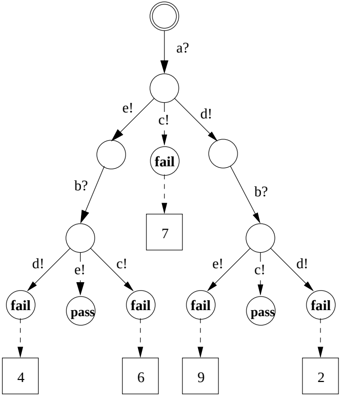
# Limitations of potential coverage



# Limitations of potential coverage




Previous work on potential coverage:
$absCov_p(f, t) = 28$

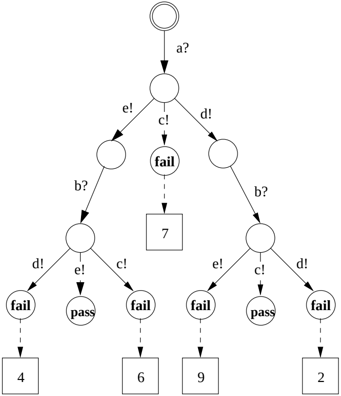



Previous work on potential coverage:
$absCov_p(f, t) = 28$

## Limitations of potential coverage

- Errors that are *potentially* covered

# Limitations of potential coverage

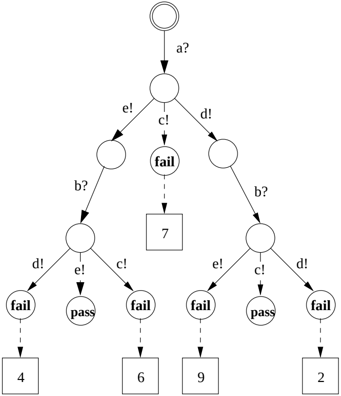



Previous work on potential coverage:
$absCov_p(f, t) = 28$

**Limitations of potential coverage**

- Errors that are *potentially* covered
- All these errors are not actually covered in every execution

# Limitations of potential coverage

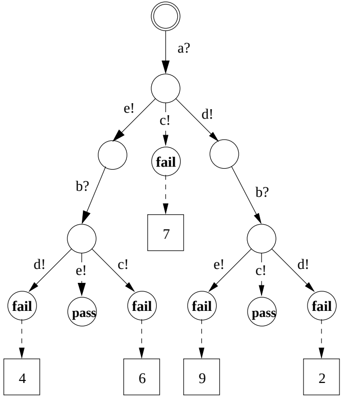



Previous work on potential coverage:
$absCov_p(f, t) = 28$

**Limitations of potential coverage**

- Errors that are *potentially* covered
- All these errors are not actually covered in every execution
- What if the test case is executed multiple times?

# Limitations of potential coverage




Previous work on potential coverage:
$absCov_p(f, t) = 28$

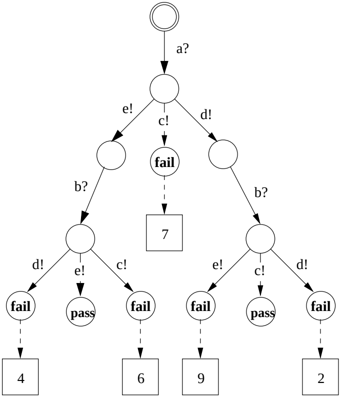
## Limitations of potential coverage

- Errors that are *potentially* covered
- All these errors are not actually covered in every execution
- What if the test case is executed multiple times?

## Actual coverage

- What is actually covered

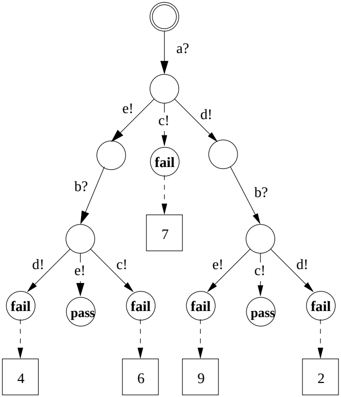
# Limitations of potential coverage




## Actual coverage

- Execution coverage:
  Faults covered when observing a specific execution

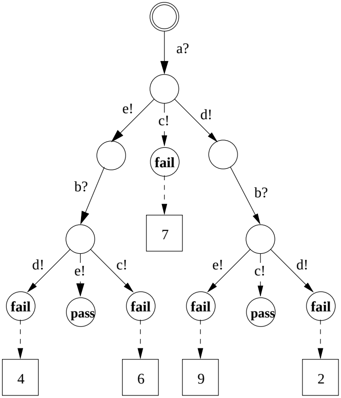
# Limitations of potential coverage




## Actual coverage

- Execution coverage:
  Faults covered when observing a specific execution
- Actual coverage:
  Probability mass distribution expressing execution coverage of single or sequence of executions

# Limitations of potential coverage




## Actual coverage

- Execution coverage:
  Faults covered when observing a specific execution
- Actual coverage:
  Probability mass distribution expressing execution coverage of single or sequence of executions
- **Expected actual coverage**

# Requirements for actual coverage

**Actual coverage**

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence

# Requirements for actual coverage

**Actual coverage**

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage

# Requirements for actual coverage

**Actual coverage**

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage
- For $n \to \infty$ executions, equal to potential coverage

# Requirements for actual coverage

**Actual coverage**

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage
- For $n \rightarrow \infty$ executions, equal to potential coverage
- Observing an error: total coverage

# Requirements for actual coverage

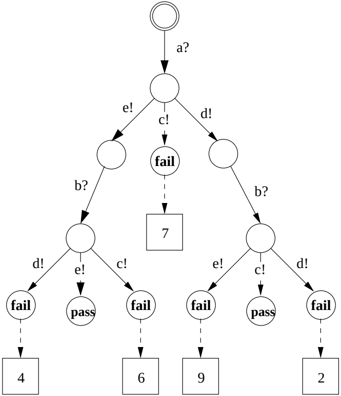
## Actual coverage

Probability mass distribution expressing execution coverage of single or sequence of executions

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage
- For $n \rightarrow \infty$ executions, equal to potential coverage
- Observing an error: total coverage
- *Not* observing an error: increase of coverage, yet no total coverage

# Contents

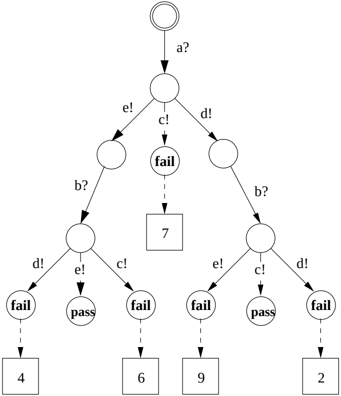
# Motivation for the model




**Actual coverage:**

Which errors will actually be covered?

# Motivation for the model

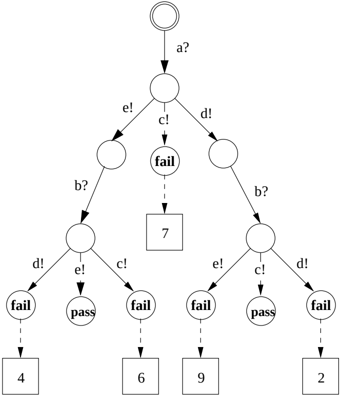



**Actual coverage:**
Which errors will actually be covered?

Necessary:

- Probabilistic transition behaviour

# Motivation for the model

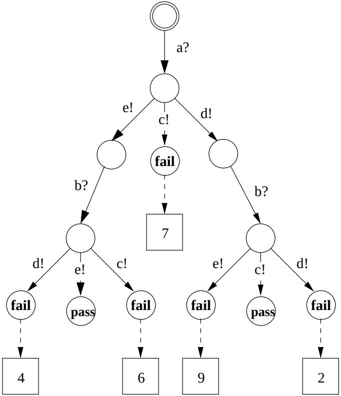



**Actual coverage:**
Which errors will actually be covered?

Necessary:

- Probabilistic transition behaviour
- Occurrence probabilities

# Motivation for the model




**Actual coverage:**
Which errors will actually be covered?

Necessary:

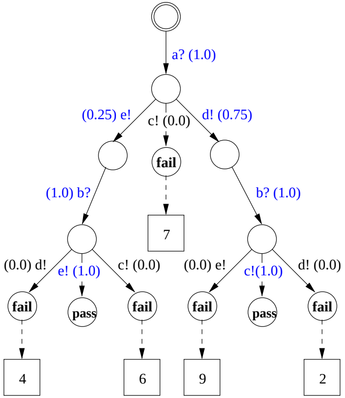
- Probabilistic transition behaviour
- Occurrence probabilities

Approach:

- Probabilities of correct outputs
- Probabilities of the presence of errors
- Probabilities of the occurrence of erroneous behaviour

# Contents

# Probabilities of correct outputs




## Definition of the correctness probability function

Correctness probability function:

- 0 for incorrect outputs
- 0 for transitions not included in the test case

Values known from implementation or measured.

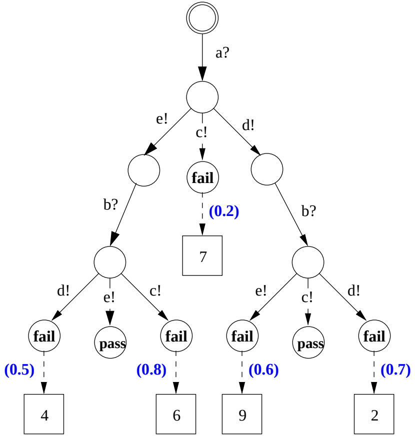
# Probabilities of the presence and occurrence of errors

**Fault presence function**

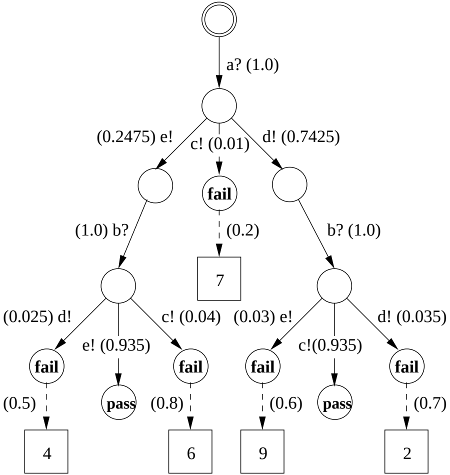
Gives the probability that a certain error is made

**Error occurrence function**

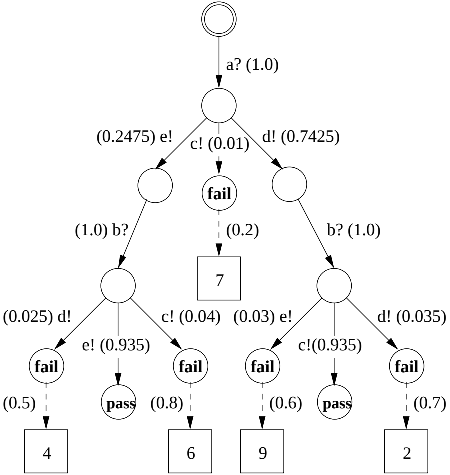
Gives the probability that a certain error occurs, *given its presence*



# Probabilistic transition behaviour



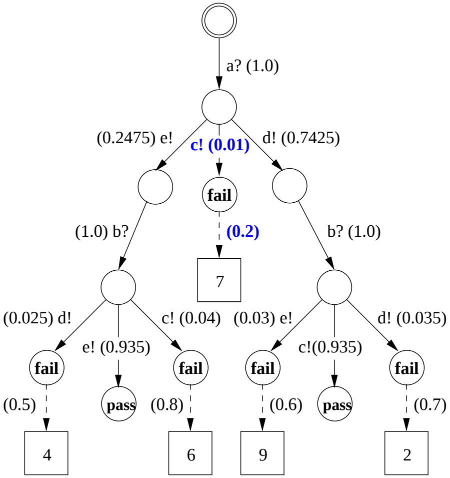
# Probabilistic transition behaviour




Erroneous outputs:

$p = p_f \times p_o$

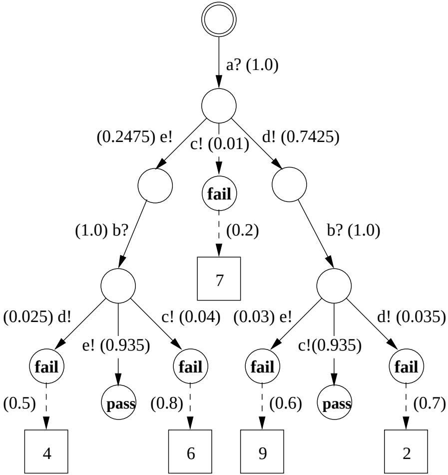
# Probabilistic transition behaviour




Erroneous outputs:

$p = p_f \times p_o$

# Probabilistic transition behaviour



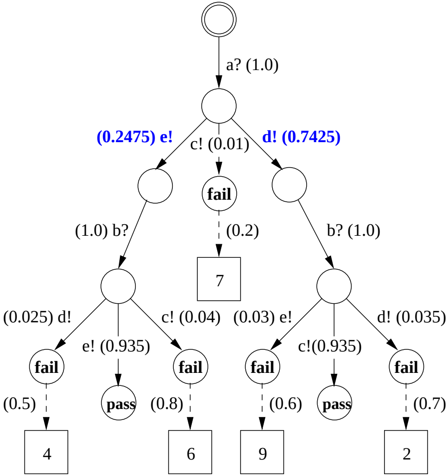

Erroneous outputs:

$p = p_f \times p_o$

Correct outputs:

$p = p_c \times (1 - \sum p_{error})$

# Probabilistic transition behaviour



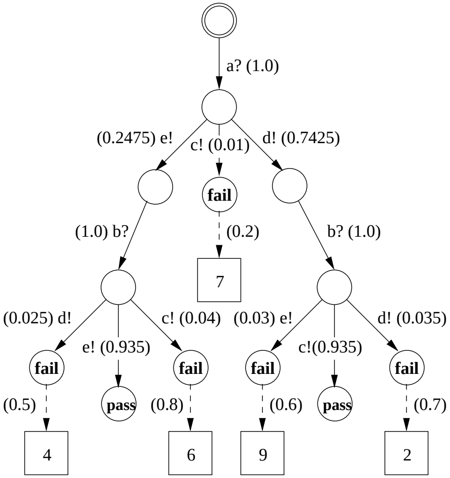

Erroneous outputs:

$p = p_f \times p_o$

Correct outputs:

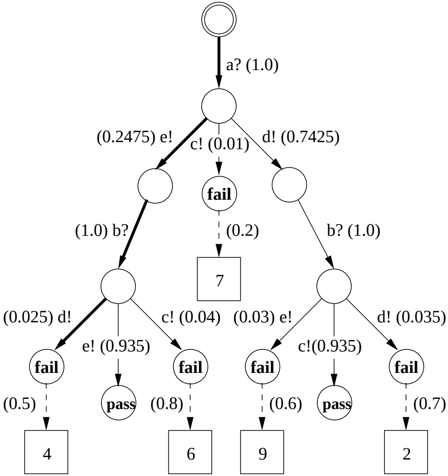
$p = p_c \times (1 - \sum p_{error})$

# Path probabilities



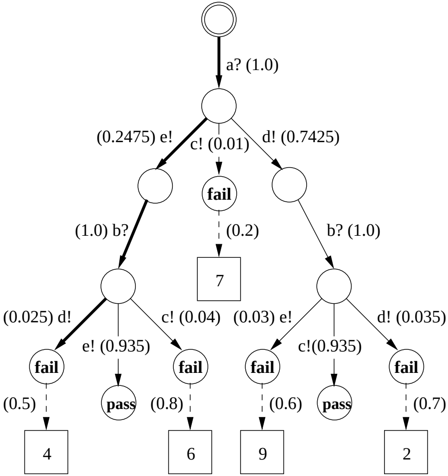

$$p(a?\ e!\ b?)(d!) = 0.025$$

# Path probabilities




$$p(a?\ e!\ b?)(d!) = 0.025$$

# Path probabilities

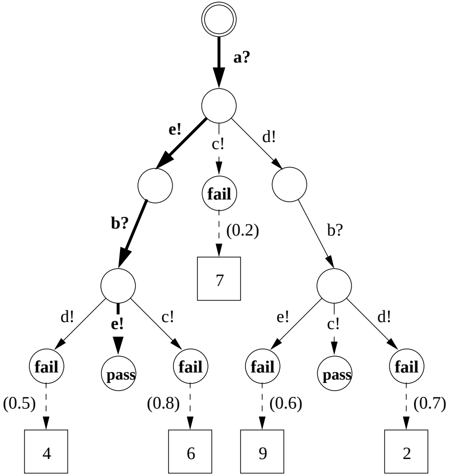



$p(a?\ e!\ b?)(d!) = 0.025$

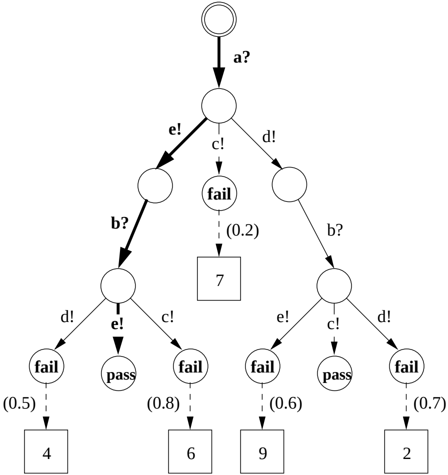
$\bar{p}(a?\ e!\ b?\ d!) =$
$1.0 \cdot 0.2475 \cdot 1.0 \cdot 0.025 = 0.006$

# Contents

# Coverage fraction

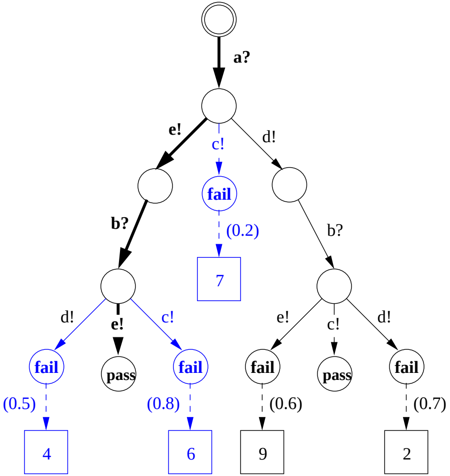


# Coverage fraction



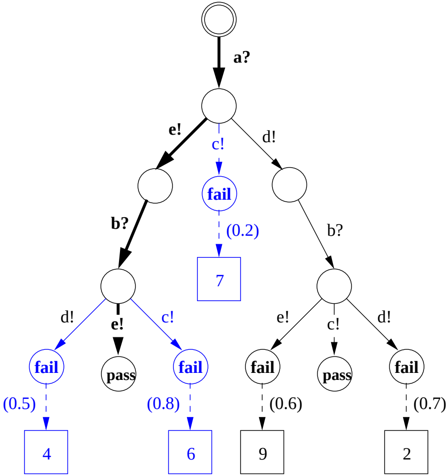

- An execution *covers* an error if it passes it.

# Coverage fraction



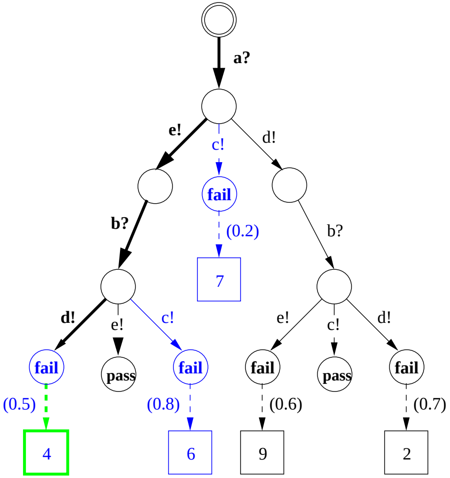

- An execution *covers* an error if it passes it.

# Coverage fraction



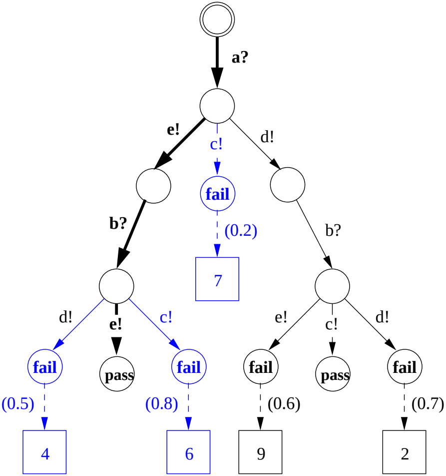

- An execution *covers* an error if it passes it.
- *Coverage fraction*: the confidence in our knowledge.

# Coverage fraction




- An execution *covers* an error if it passes it.
- *Coverage fraction*: the confidence in our knowledge.
- Observing an error yields total certainty: $\mathit{CovFrac} = 1$.

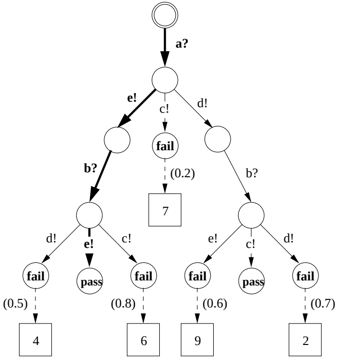
# Coverage fraction




- An execution *covers* an error if it passes it.
- *Coverage fraction*: the confidence in our knowledge.
- Observing an error yields total certainty: $CovFrac = 1$.
- *Not* observing an error $n$ times: $CovFrac = 1 - (1 - p_o)^n$

# Contents

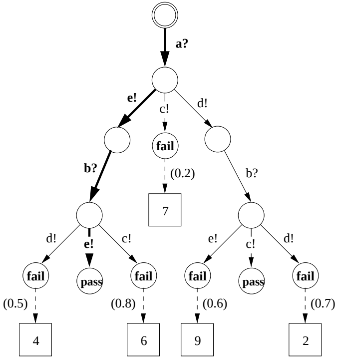
# Execution coverage




**Def. of execution coverage**

$$absExCov(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \mathrm{CovFrac}(\sigma', p_o, \sigma)$$

# Execution coverage

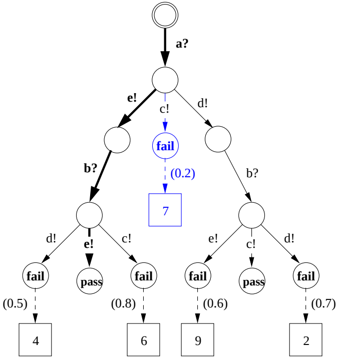



**Def. of execution coverage**

$$absExCov(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \mathrm{CovFrac}(\sigma', p_o, \sigma)$$

absExCov(..) $=$

# Execution coverage

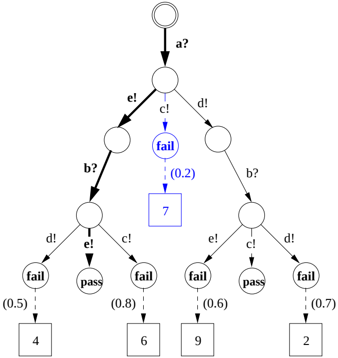



absExCov(..) $=$

**Def. of execution coverage**

$$absExCov(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \mathrm{CovFrac}(\sigma', p_o, \sigma)$$

# Execution coverage

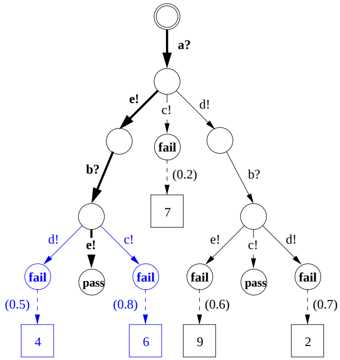



**Def. of execution coverage**

$$absExCov(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \text{CovFrac}(\sigma', p_o, \sigma)$$

$$\text{absExCov}(..) = 7 \cdot (1 - (1 - 0.2)^1) +$$

# Execution coverage

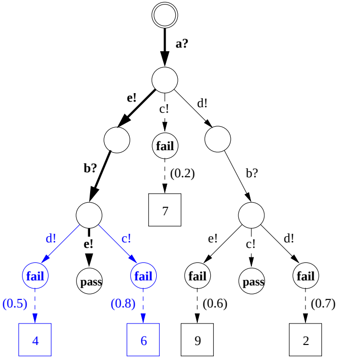



**Def. of execution coverage**

$$absExCov(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \mathrm{CovFrac}(\sigma', p_o, \sigma)$$

$$\mathrm{absExCov}(..) = 7 \cdot (1 - (1 - 0.2)^1) +$$

# Execution coverage

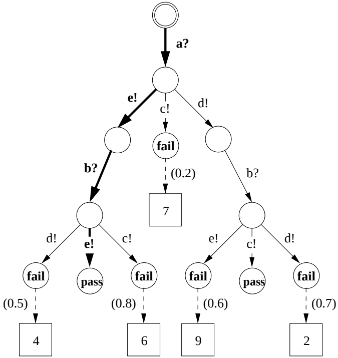



**Def. of execution coverage**

$$absExCov(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \mathrm{CovFrac}(\sigma', p_o, \sigma)$$

$$\mathrm{absExCov}(..) = 7 \cdot (1 - (1 - 0.2)^1) + 4 \cdot 0.5 + 6 \cdot 0.8 = 8.2$$

# Execution coverage




**Def. of execution coverage**

$$absExCov(\sigma, t, f, p_o) = \sum_{\sigma' \in t} f(\sigma') \cdot \mathrm{CovFrac}(\sigma', p_o, \sigma)$$

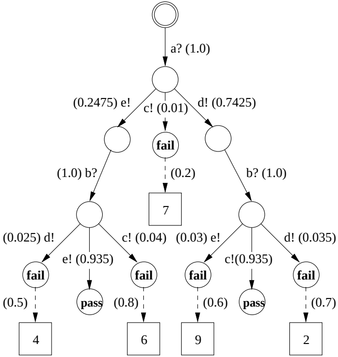
$$\mathrm{absExCov}(..) = 7 \cdot (1 - (1 - 0.2)^1) + 4 \cdot 0.5 + 6 \cdot 0.8 = 8.2$$

For three times this execution:

$$\mathrm{absExCov}(..) = 7 \cdot (1 - (1 - 0.2)^3) + \cdots = 12.868$$

# Contents

# Actual coverage of test cases

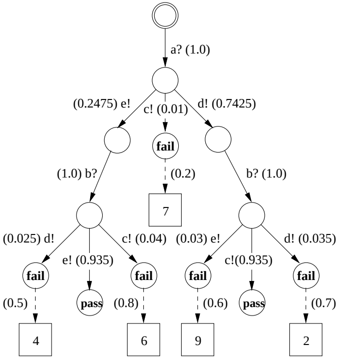



## Actual coverage of a single execution

The actual coverage of a single execution is a random variable.

$$\mathbb{P}[absCov^{\text{single}}_{t,f,p,p_o} = x] = \sum_{\substack{\sigma \in \text{exec}_t \\ \text{absExCov}(\sigma,t,f,p_o)=x}} \bar{p}(\sigma)$$

# Actual coverage of test cases




## Actual coverage of a sequence of executions

The actual coverage of a sequence of execution is also a random variable.

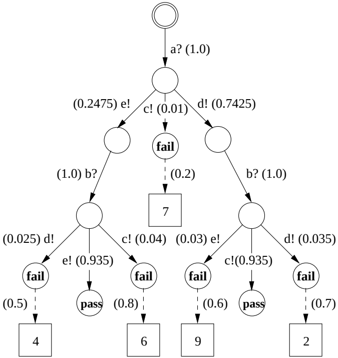
$$\mathbb{P}[absCov^n_{t,f,p,p_o} = x] = \sum_{\substack{E \in \mathrm{exec}^n_t \\ \mathrm{absExCov}(E,t,f,p_o)=x}} \bar{p}(E)$$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in \text{exec}_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in exec_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$

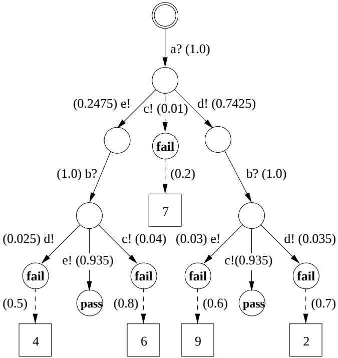



**Potential coverage**

Absolute potential coverage: 28

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in \mathrm{exec}_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$




## Potential coverage

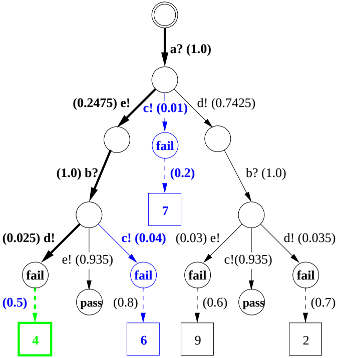
Absolute potential coverage: 28

## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) =$
$absExCov(a?\ e!\ b?\ d!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ d!) +$
$absExCov(a?\ e!\ b?\ e!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ e!) +$
$absExCov(a?\ e!\ b?\ c!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ c!) +$
$\cdots + absExCov(a?\ c!, t, f, p_o) \cdot \bar{p}(a?\ c!)$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in \mathrm{exec}_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$




## Potential coverage

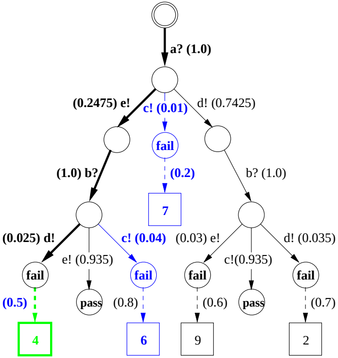
Absolute potential coverage: 28

## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) =$
$absExCov(a?\ e!\ b?\ d!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ d!) +$
$absExCov(a?\ e!\ b?\ e!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ e!) +$
$absExCov(a?\ e!\ b?\ c!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ c!) +$
$\cdots + absExCov(a?\ c!, t, f, p_o) \cdot \bar{p}(a?\ c!)$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in \mathrm{exec}_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$




## Potential coverage

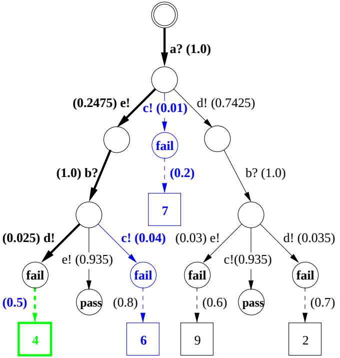
Absolute potential coverage: 28

## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) =$
$(7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot \bar{p}(a?\ e!\ b?\ d!) +$
$absExCov(a?\ e!\ b?\ e!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ e!) +$
$absExCov(a?\ e!\ b?\ c!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ c!) +$
$\cdots + absExCov(a?\ c!, t, f, p_o) \cdot \bar{p}(a?\ c!)$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in exec_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$




## Potential coverage

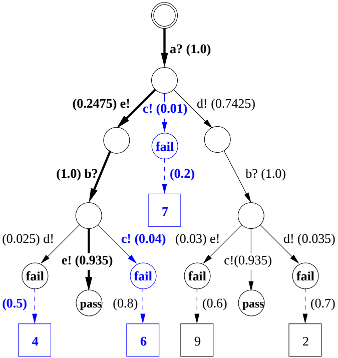
Absolute potential coverage: 28

## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) =$
$(7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) +$
$absExCov(a?\ e!\ b?\ e!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ e!) +$
$absExCov(a?\ e!\ b?\ c!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ c!) +$
$\cdots + absExCov(a?\ c!, t, f, p_o) \cdot \bar{p}(a?\ c!)$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in \mathrm{exec}_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$




## Potential coverage

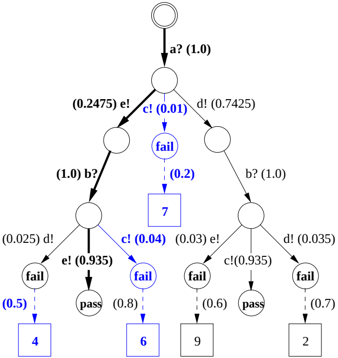
Absolute potential coverage: 28

## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) =$
$(7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) +$
$absExCov(a?\ e!\ b?\ e!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ e!) +$
$absExCov(a?\ e!\ b?\ c!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ c!) +$
$\cdots + absExCov(a?\ c!, t, f, p_o) \cdot \bar{p}(a?\ c!)$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in \mathrm{exec}_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$




## Potential coverage

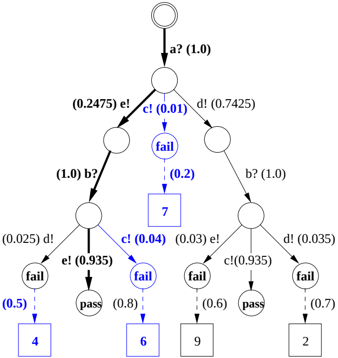
Absolute potential coverage: 28

## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) =$
$(7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) +$
$(7 \cdot 0.2 + 4 \cdot 0.5 + 6 \cdot 0.8) \cdot \bar{p}(a?\ e!\ b?\ e!) +$
$absExCov(a?\ e!\ b?\ c!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ c!) +$
$\cdots + absExCov(a?\ c!, t, f, p_o) \cdot \bar{p}(a?\ c!)$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in \mathrm{exec}_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$




## Potential coverage

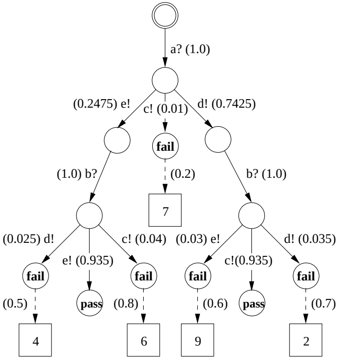
Absolute potential coverage: 28

## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) =$
$(7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) +$
$(7 \cdot 0.2 + 4 \cdot 0.5 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.935) +$
$absExCov(a?\ e!\ b?\ c!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ c!) +$
$\cdots + absExCov(a?\ c!, t, f, p_o) \cdot \bar{p}(a?\ c!)$

# Expected actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = \sum_{\sigma \in \mathrm{exec}_t} absExCov(\sigma, t, f, p_o) \cdot \bar{p}(\sigma)$$




## Potential coverage

Absolute potential coverage: 28

## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) =$
$(7 \cdot 0.2 + 4 \cdot 1 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.025) +$
$(7 \cdot 0.2 + 4 \cdot 0.5 + 6 \cdot 0.8) \cdot (0.2475 \cdot 0.935) +$
$absExCov(a?\ e!\ b?\ c!, t, f, p_o) \cdot \bar{p}(a?\ e!\ b?\ c!) +$
$\cdots + absExCov(a?\ c!, t, f, p_o) \cdot \bar{p}(a?\ c!) = 8.3$

# Expected actual coverage

**Expected value of the actual coverage for a sequence of executions**

$$E(absCov^n_{t,f,p,p_o}) = \sum_{E \in \mathrm{exec}^n_t} absExCov(E, t, f, p_o) \cdot \bar{p}(E)$$

# Expected actual coverage

**Expected value of the actual coverage for a sequence of executions**

$$E(absCov^{n}_{t,f,p,p_o}) = \sum_{E \in \text{exec}^n_t} absExCov(E, t, f, p_o) \cdot \bar{p}(E)$$

Problem: exponential in $n$, so not very feasible in practice.

# Expected actual coverage

I found a solution:

**Expected value of the actual coverage for a sequence of executions**

$$E(absCov^n_{t,f,p,p_o}) =$$
$$\sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n) \cdot 1 + \right.\right.$$
$$\left.\left. \sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \right)\right)$$

# Expected actual coverage

I found a solution:

**Expected value of the actual coverage for a sequence of executions**

$$E(absCov^n_{t,f,p,p_o}) =$$

$$\sum_{\sigma a \in t} \left( \mathbf{f(\sigma a)} \cdot \left( \mathbf{(1 - (1 - \bar{p}(\sigma} a\mathbf{))^n) \cdot 1} + \right.\right.$$

$$\left.\left. \sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \right)\right)$$

# Expected actual coverage

I found a solution:

**Expected value of the actual coverage for a sequence of executions**

$$E(absCov^n_{t,f,p,p_o}) =$$
$$\sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n) \cdot 1 + \right.\right.$$
$$\left.\left. \sum_{\mathbf{i=0}}^{\mathbf{n}} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \right) \right)$$

# Expected actual coverage

I found a solution:

**Expected value of the actual coverage for a sequence of executions**

$$E(absCov^n_{t,f,p,p_o}) =$$
$$\sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n) \cdot 1 + \sum_{i=0}^{n} \binom{\mathbf{n}}{\mathbf{i}} \bar{\mathbf{p}}(\boldsymbol{\sigma})^{\mathbf{i}} (\mathbf{1} - \bar{\mathbf{p}}(\boldsymbol{\sigma}))^{\mathbf{n}-\mathbf{i}} (1 - p(\sigma, a))^i (1 - (1 - p_o(\sigma, a))^i) \right) \right)$$

# Expected actual coverage

I found a solution:

**Expected value of the actual coverage for a sequence of executions**

$$E(absCov^n_{t,f,p,p_o}) =$$
$$\sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n) \cdot 1 + \right.\right.$$
$$\left.\left. \sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} \mathbf{(1 - p(\boldsymbol{\sigma}, a))^i} (1 - (1 - p_o(\sigma, a))^i) \right) \right)$$

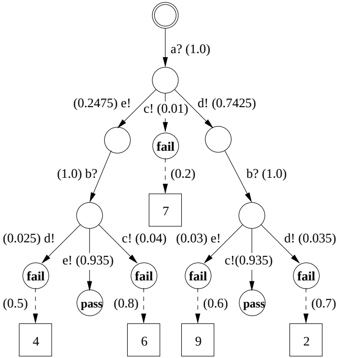
# Expected actual coverage

I found a solution:

**Expected value of the actual coverage for a sequence of executions**

$$E(absCov^n_{t,f,p,p_o}) = \sum_{\sigma a \in t} \left( f(\sigma a) \cdot \left( (1 - (1 - \bar{p}(\sigma a))^n) \cdot 1 + \sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a))^i \mathbf{(1 - (1 - p_o(\sigma, a))^i)} \right) \right)$$

# Example of actual coverage




## Actual coverage

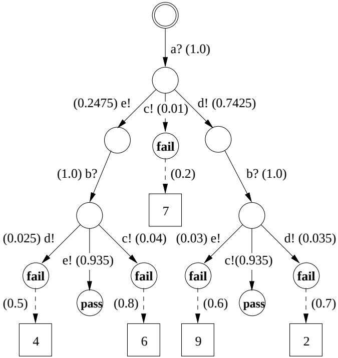
$E(absCov^{single}_{t,f,p,p_o}) = 8.3$

## Potential coverage

Absolute potential coverage: 28

## Actual coverage

$E(absCov^{5}_{t,f,p,p_o}) =$




## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) = 8.3$

## Potential coverage

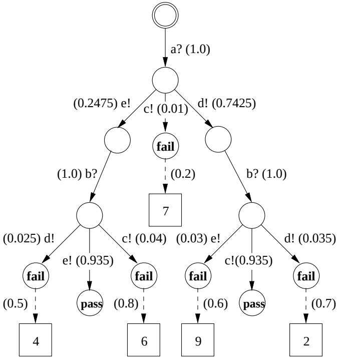
Absolute potential coverage: 28

## Actual coverage

$$E(absCov^{5}_{t,f,p,p_o}) =$$

$$7 \cdot \left( (1-(1-0.01)^5) \cdot 1 + \sum_{i=0}^{5} \left( \binom{5}{i} 1^i \cdot 0^{5-i} \cdot (1-0.01)^i \cdot (1-(1-0.2)^i) \right) \right)$$

# Example of actual coverage




## Actual coverage

$$E(absCov^{single}_{t,f,p,p_o}) = 8.3$$

## Potential coverage

Absolute potential coverage: 28

## Actual coverage

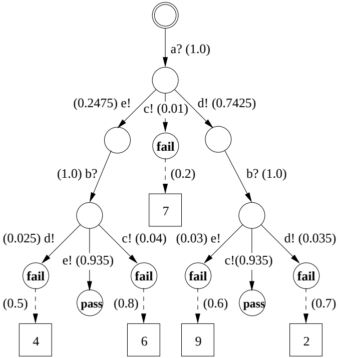
$$E(absCov^{5}_{t,f,p,p_o}) =$$

$$7 \cdot \Bigg( (1-(1-0.01)^5) \cdot 1 + \sum_{i=0}^{5} \Big( \binom{5}{i} 1^i \cdot 0^{5-i} \cdot (1-0.01)^i \cdot (1-(1-0.2)^i) \Big) \Bigg)$$

$$+\ 4 \cdot \Bigg( (1-(1-0.2475 \cdot 0.025)^5) \cdot 1 + \sum_{i=0}^{5} \Big( \binom{5}{i} 0.2475^i \cdot (1-0.2475)^{5-i} \cdot (1-0.025)^i \cdot (1-(1-0.5)^i) \Big) \Bigg)$$

$$+ \cdots$$

# Example of actual coverage




## Actual coverage

$E(absCov^{single}_{t,f,p,p_o}) = 8.3$

## Potential coverage

Absolute potential coverage: 28

## Actual coverage

$$E(absCov^5_{t,f,p,p_o}) =$$

$$7 \cdot \Bigg( (1 - (1 - 0.01)^5) \cdot 1 + \sum_{i=0}^{5} \Big( \binom{5}{i} 1^i \cdot 0^{5-i} \cdot (1 - 0.01)^i \cdot (1 - (1 - 0.2)^i) \Big) \Bigg)$$

$$+\ 4 \cdot \Bigg( (1 - (1 - 0.2475 \cdot 0.025)^5) \cdot 1 + \sum_{i=0}^{5} \Big( \binom{5}{i} 0.2475^i \cdot (1 - 0.2475)^{5-i} \cdot (1 - 0.025)^i \cdot (1 - (1 - 0.5)^i) \Big) \Bigg)$$

$$+ \cdots = 21.45$$

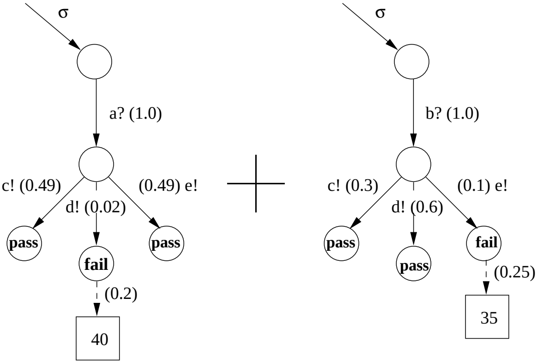
# Asymptotical behaviour

**Theorem**

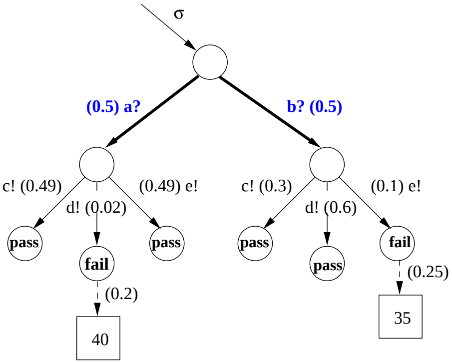
$$\lim_{n \to \infty} E(absCov^n_{t,f,p,p_o}) = absCov_p(t, f)$$

# Contents

# Actual coverage of test suites

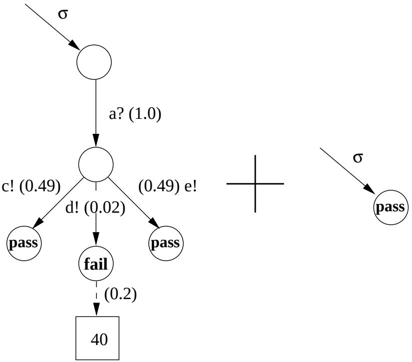


# Actual coverage of test suites



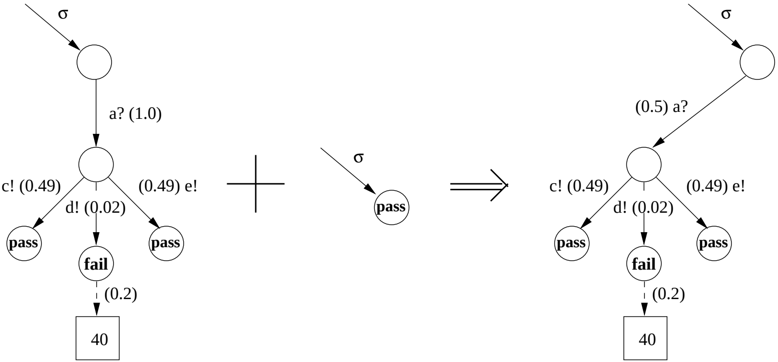

$|T| \cdot n$ executions of supertest $\equiv$ $n$ executions of test suite

# Actual coverage of test suites – different depth



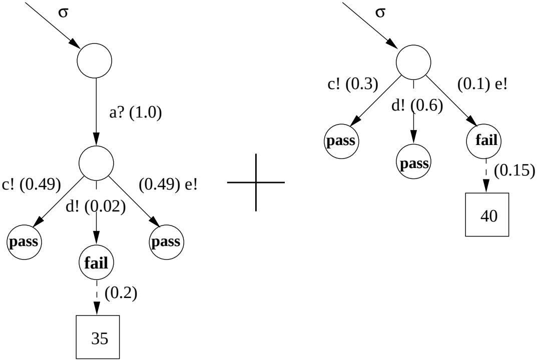

$|T| \cdot n$ executions of supertest $\equiv$ $n$ executions of test suite

# Actual coverage of test suites – different depth



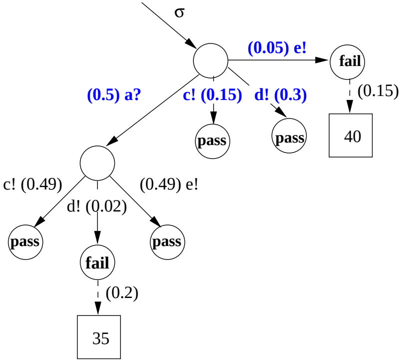

$|T| \cdot n$ executions of supertest $\equiv$ $n$ executions of test suite

# Actual coverage of test suites – input vs. output



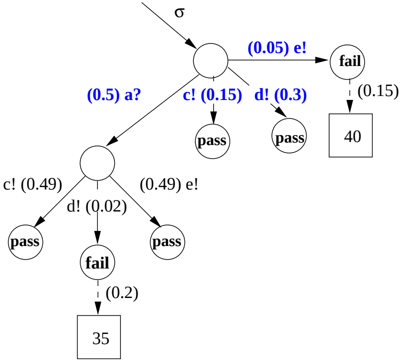

$|T| \cdot n$ executions of supertest $\equiv$ $n$ executions of test suite

# Actual coverage of test suites – input vs. output




$|T| \cdot n$ executions of supertest $\equiv$ $n$ executions of test suite

# Actual coverage of test suites – input vs. output




$|T| \cdot n$ executions of supertest $\equiv$ $n$ executions of test suite

Problem: $\sigma e!$ seems to be covered all the time

# Actual coverage of test suites

$$E(absCov^n_{t,f,p,p_o}) = \sum_{\sigma a \in t} \Bigg( f(\sigma a) \cdot \Bigg( (1 - (1 - \bar{p}(\sigma a))^n) \cdot 1 +$$

$$\sum_{i=0}^{n} \binom{n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{n-i} (1 - p(\sigma, a)^i)(1 - (1 - p_o(\sigma, a))^i) \Bigg) \Bigg)$$

# Actual coverage of test suites

$$E(absCov^n_{t,f,p,p_o}) = \sum_{\sigma a \in t} \Bigg( f(\sigma a) \cdot \Bigg( (1 - (1 - \bar{p}(\sigma a))^{\mathbf{c}(\boldsymbol{\sigma})n}) \cdot 1 +$$

$$\sum_{i=0}^{\mathbf{c}(\boldsymbol{\sigma})n} \binom{\mathbf{c}(\boldsymbol{\sigma})n}{i} \bar{p}(\sigma)^i (1 - \bar{p}(\sigma))^{\mathbf{c}(\boldsymbol{\sigma})n-i} (1 - p(\sigma, a)^i)(1 - (1 - p_o(\sigma, a))^i) \Bigg) \Bigg)$$

$c(\sigma)$: the fraction of test cases that observe after $\sigma$.

# Contents

# Conclusions

- Defined a quiescence-preserving transformation of non-deterministic fault automata to deterministic fault automata

# Conclusions

- Defined a quiescence-preserving transformation of non-deterministic fault automata to deterministic fault automata
- Developed a specification mechanism for probabilistic transition behaviour

# Conclusions

- Defined a quiescence-preserving transformation of non-deterministic fault automata to deterministic fault automata
- Developed a specification mechanism for probabilistic transition behaviour
- Developed a notion of actual test coverage which applies to test cases *and* test suites, with polynomially computable expectations

# Conclusions

- Defined a quiescence-preserving transformation of non-deterministic fault automata to deterministic fault automata
- Developed a specification mechanism for probabilistic transition behaviour
- Developed a notion of actual test coverage which applies to test cases *and* test suites, with polynomially computable expectations

- Indication of confidence in our knowledge on error presence
- Include number of executions. More executions, more coverage
- For $n \rightarrow \infty$ executions, equal to potential coverage
- Observing an error: total coverage
- *Not* observing an error: increase of coverage, yet no total coverage

# Contents

# Future work

- Test evaluation
  - Given results of executions, draw conclusions

# Future work

- Test evaluation
  - Given results of executions, draw conclusions
- Test optimization
  - Construct test case of size $k$ with maximum coverage

# Future work

- Test evaluation
  - Given results of executions, draw conclusions
- Test optimization
  - Construct test case of size $k$ with maximum coverage
  - Construct smallest test suite with $p$ percent actual relative coverage

# Future work

- Test evaluation
  - Given results of executions, draw conclusions
- Test optimization
  - Construct test case of size $k$ with maximum coverage
  - Construct smallest test suite with $p$ percent actual relative coverage
  - ...