UNIVERSITY OF TWENTE.

Formal Methods & Tools.

A linear process algebraic format for probabilistic systems

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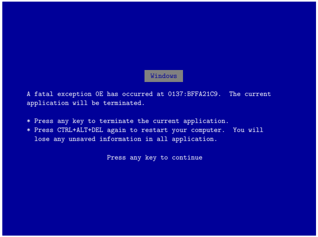
Joint work with Joost-Pieter Katoen, Jaco van de Pol, and Mariëlle Stoelinga

FMT Lunchmeeting

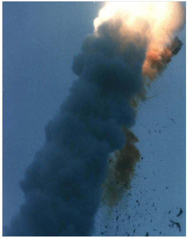
Contents

Introduction – Dependability

Dependability of computer systems is becoming more and more important.



Windows blue screen



Ariane 5 crash

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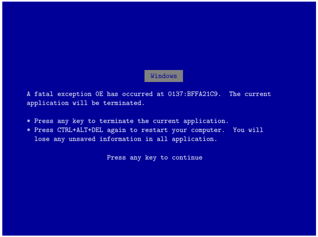

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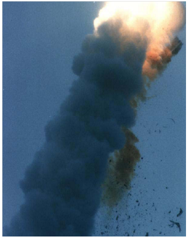
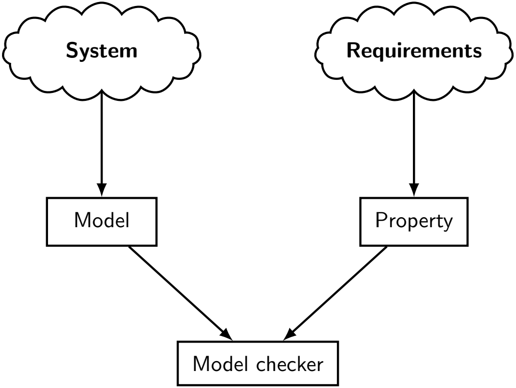


Ariane 5 crash

Our aim: use quantitative formal methods to improve system quality.

Introduction – Model Checking

A popular solution is model checking; verifying properties of a system by constructing a model and ranging over its state space.



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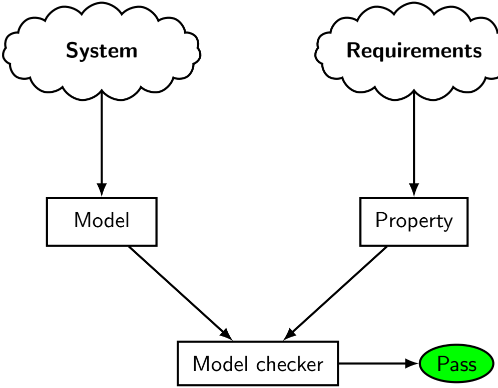

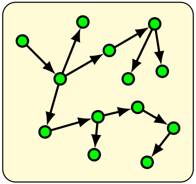

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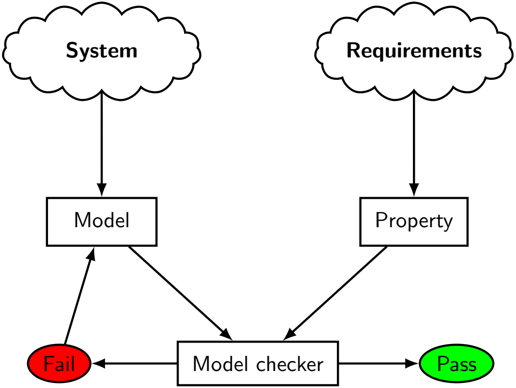

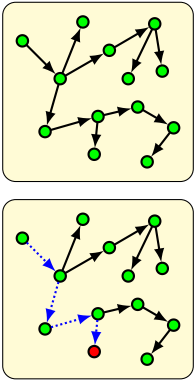

Introduction – Probabilistic Model Checking

Probabilistic model checking:

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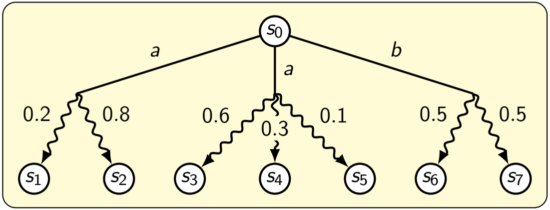

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- Probabilistically choose the next state

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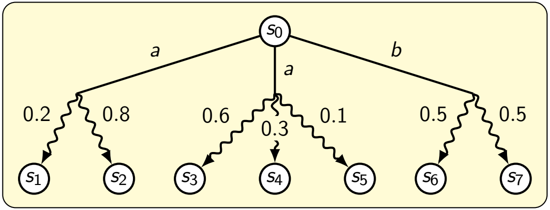

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Applications:

- Dependability analysis
- Performance analysis

Introduction – Probabilistic Model Checking

Limitations of previous approaches:

- Susceptible to the state space explosion problem
- Restricted treatment of data

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Our approach:

1. Define a probabilistic process algebra (prCRL), incorporating both data types and probabilistic choice
2. Define a linear format (the LPPE), enabling symbolic optimisations at the language level
3. Develop and implement a linearisation algorithm
4. Reduce state spaces before they are generated by manipulations of the linear format.

Strong probabilistic bisimulation

Equivalent PAs: strongly probabilistic bisimilar PAs

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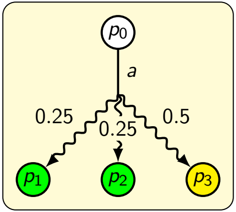

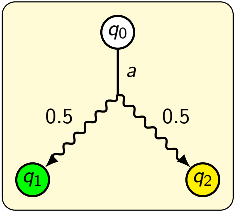

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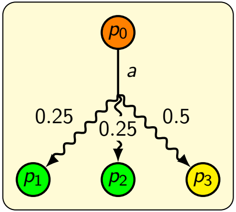
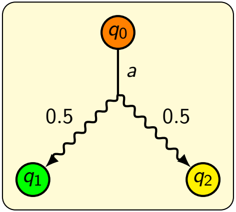
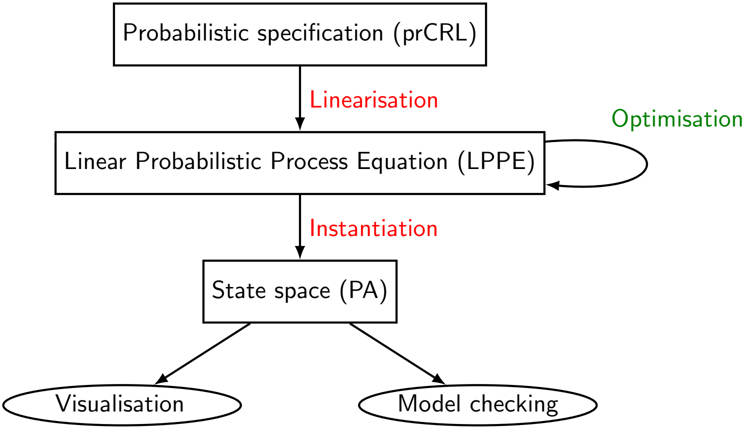




Introduction – overview of our approach



A process algebra with data and probability: prCRL

Specification language prCRL:

- Based on μCRL (so data), with additional probabilistic choice
- Operational semantics defined in terms of probabilistic automata
- Minimal set of operators to facilitate formal manipulation
- Syntactic sugar easily definable

A process algebra with data and probability: prCRL

The grammar of prCRL process terms

Process terms in prCRL are obtained by the following grammar:

$$p ::= Y(\vec{t}) \mid c \Rightarrow p \mid p + p \mid \sum_{x:D} p \mid a(\vec{t}) \sum_{x:D}\!\!\bullet\, f : p$$

- c is a condition (boolean expression)
- a is an atomic action
- f is a real-valued expression yielding values in $[0, 1]$
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Process equations and processes

A process equation is something of the form $X(\vec{g} : \vec{G}) = p$.

Some examples

Sending an arbitrary natural number

$$X = \tau \sum_{n:\mathbb{N}^{>0}} \frac{1}{2^n} : (\mathsf{send}(n) \cdot \sum_{j:\{*\}} 1.0 : X)$$

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Sending ping messages until system crash

$$X = \mathrm{ping} \sum_{i:\{1,2\}} \bullet (i = 1 \ ? \ 0.1 \ : \ 0.9) : ((i = 1 \Rightarrow \mathrm{crash}) + (i \neq 1 \Rightarrow X))$$

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Writing all Fibonacci numbers

$$X(p : \mathbb{N}, pp : \mathbb{N}) = \mathsf{write}(\mathsf{plus}(p, pp)) \cdot X(\mathsf{plus}(p, pp), p)$$

Operational semantics

$$\textsc{NChoice-L}\ \frac{p \xrightarrow{\alpha} \mu}{p + q \xrightarrow{\alpha} \mu}$$

$$\textsc{Implies}\ \frac{p \xrightarrow{\alpha} \mu}{c \Rightarrow p \xrightarrow{\alpha} \mu}\ \text{if } c \text{ holds}$$

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An arbitrary specification

$$\begin{aligned} X(x : \text{Bool}) = x \quad & \Rightarrow a \cdot X(x) + b \cdot X(\text{not}(x)) \\ + \text{not}(x) & \Rightarrow c(0.5 : X(\text{false}) \oplus 0.5 : X(\text{true})) \end{aligned}$$

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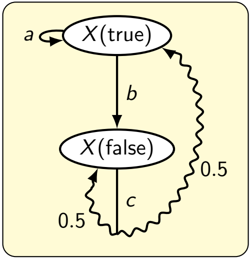

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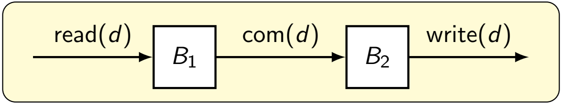
Linear process equations

In the non-probabilistic setting, LPEs are given by

$$X(\vec{g} : \vec{G}) = \sum_{\vec{d_1}:\vec{D_1}} c_1 \Rightarrow a_1(b_1) \cdot X(\vec{n_1})$$
$$\dots$$
$$+ \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k(b_k) \cdot X(\vec{n_k})$$

- $\vec{G}$ is a type for state vectors
- $\vec{D_i}$ a type for local variable vectors for summand i
- c_i is the enabling condition of summand i
- a_i is an atomic action, with action-parameter vector b_i
- $\vec{n_i}$ is the next-state vector of summand i.

Linear process equations – An example



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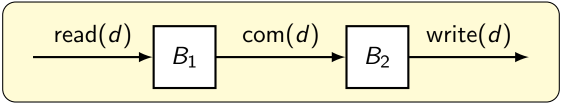

$$B_1 = \sum_{d:D} \text{read}(d) \cdot \text{com}(d) \cdot B_1$$

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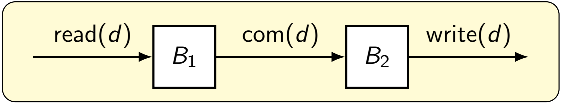

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$$\begin{aligned}
X(a : \{1,2\}, b : \{1,2\}, x : D, y : D) = \\
\textstyle\sum_{d:D} \quad & a = 1 && \Rightarrow \mathrm{read}(d) \cdot X(2, b, d, y) && (1)\\
+ \quad & a = 2 \wedge b = 1 && \Rightarrow \mathrm{com}(x) \cdot X(1, 2, x, x) && (2)\\
+ \quad & b = 2 && \Rightarrow \mathrm{write}(y) \cdot X(a, 1, x, y) && (3)
\end{aligned}$$

A linear format for prCRL: the LPPE

In the probabilistic setting, LPPEs are given by

$$X(\vec{g} : \vec{G}) = \sum_{\vec{d_1}:\vec{D_1}} c_1 \Rightarrow a_1(b_1) \sum\nolimits^{\bullet}_{\vec{e_1}:\vec{E_1}} f_1 : X(\vec{n_1})$$

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& \ldots \\
& + \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k(b_k) \sum\!\!\!\!\bullet_{\vec{e_k}:\vec{E_k}} f_k : X(\vec{n_k})
\end{aligned}
$$

Advantages of using LPPEs instead of prCRL specifications:

- Easy state space generation
- Straight-forward parallel composition
- **Symbolic optimisations enabled at the language level**

A linear format for prCRL: the LPPE

An example

$$X(\mathrm{pc} : \{1, 2\}, d : D) = \mathrm{pc} = 1 \Rightarrow \tau \sum_{e:D}^{\bullet} \frac{1}{|D|} : X(2, e)$$
$$+ \mathrm{pc} = 2 \Rightarrow \mathrm{send}(d) \cdot X(1, d))$$

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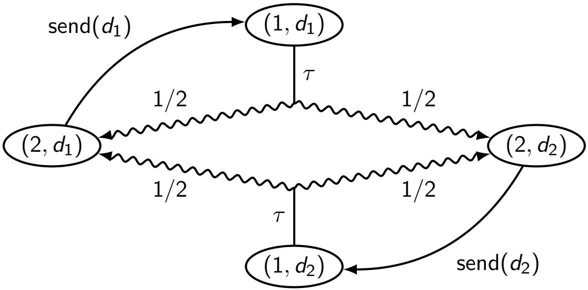

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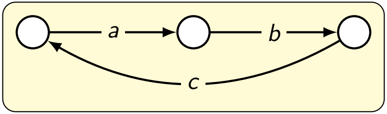

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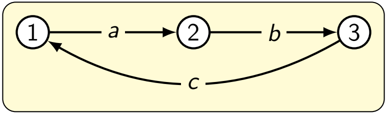

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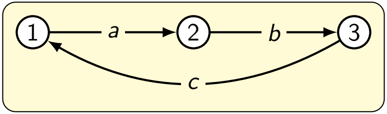

$$\begin{aligned} Y(pc\colon \{1,2,3\}) = & \\ & pc = 1 \Rightarrow a \cdot Y(2) \\ + & pc = 2 \Rightarrow b \cdot Y(3) \\ + & pc = 3 \Rightarrow c \cdot Y(1) \end{aligned}$$

Linearisation

$$X = \sum_{d:D} \mathrm{in}(d) \cdot (\tau \cdot \mathrm{loss} \cdot X + \tau \cdot \mathrm{out}(d) \cdot X)$$

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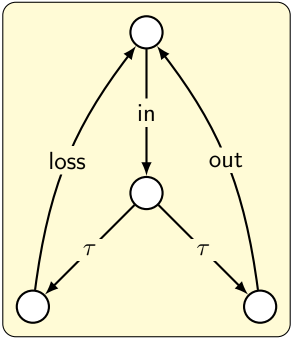

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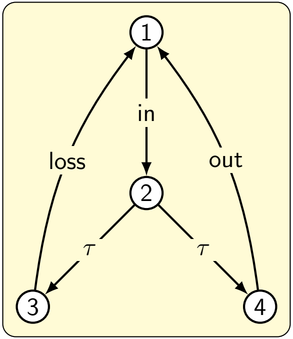

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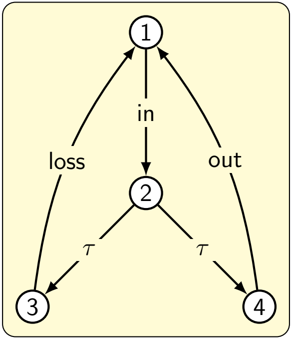

$$\begin{aligned} Y(pc\colon \{1,2,3,4\}, x\colon D) = & \\ \textstyle\sum_{d:D} \quad & pc = 1 \Rightarrow \text{in}(d) \cdot Y(2,d) \\ + \quad & pc = 2 \Rightarrow \tau \cdot Y(3,x) \\ + \quad & pc = 2 \Rightarrow \tau \cdot Y(4,x) \\ + \quad & pc = 3 \Rightarrow \text{loss} \cdot Y(1,x) \\ + \quad & pc = 4 \Rightarrow \text{out}(x) \cdot Y(1,x) \end{aligned}$$

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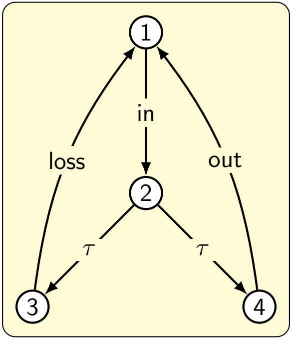

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Initial process: $Y(1, d_1)$.

Linearisation

$$X(d : D) = \sum_{e:D} a(d+e) \sum\!\!\!\!\!\!\bullet_{f:D} \ \tfrac{1}{|D|} : \left(c(e) \cdot c(f) \cdot X(5) + c(e+f) \cdot X(5) \right)$$

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---

2 $X_1(d : D, e : D, f : D) = \sum_{e:D} a(d + e) \sum\!\!\bullet_{f:D} \frac{1}{|D|} \cdot X_2(d, e, f)$

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Linearisation

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$$X(\mathrm{pc}:\{1,2,3\}, d:D, e:D, f:D) =$$
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Linearisation

In general, we always linearise in two steps:

1. Transform the specification to intermediate regular form (IRF) (every process is a summation of single-action terms)
2. Merge all processes into one big process by introducing a program counter

In the first step, global parameters are introduced to remember the values of bound variables.

Theorem

A specification S and the specification S' obtained by linearising S are strongly probabilistic bisimilar.

Extended prCRL

For compositionality we introduce extended prCRL. It extends prCRL by parallel composition, encapsulation, hiding and renaming.

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The grammar of extended prCRL process terms

Process terms in extended prCRL are obtained by:

$$q ::= p \mid q \,||\, q \mid \partial_E(q) \mid \tau_H(q) \mid \rho_R(q)$$

- $q_1 \,||\, q_2$: parallel composition with ACP-style communication
- $\partial_E(q)$: encapsulation of all actions in E
- $\tau_H(q)$: hiding of all actions in H
- $\rho_R(q)$: renaming of actions according to the function R

Operational semantics of parallel composition

$$\textsc{Par-L}\ \frac{p \xrightarrow{\alpha} \mu}{p \,||\, q \xrightarrow{\alpha} \mu'} \text{ where } \forall p' \,.\, \mu'(p' \,||\, q) = \mu(p')$$

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$$X(d : D) = \text{out}(d) \sum_{d' : D} \frac{1}{|D|} : X(d') \qquad Y(n : \mathbb{N}) = \text{write}(n) \cdot Y(n+1)$$

$$Z = X(d_1) \,||\, Y(0)$$

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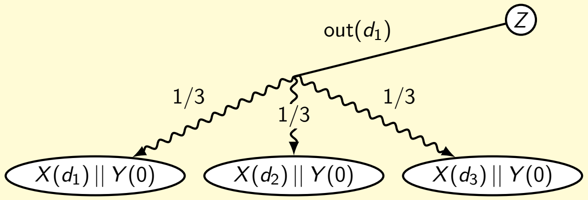

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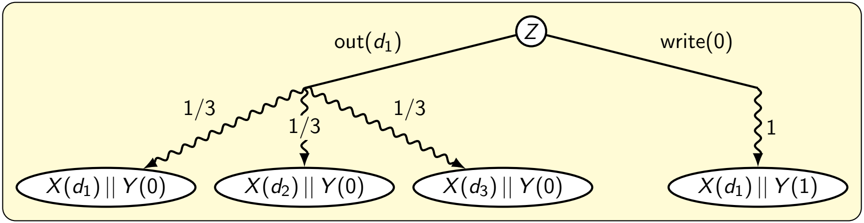

Operational semantics of parallel composition

For communication we assume a partial function

$$\gamma \colon \mathsf{Act} \times \mathsf{Act} \to \mathsf{Act}$$

When $(a, b) \in \mathrm{dom}(\gamma)$, communication between a and b yields $\gamma(a, b)$.

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$$\frac{p \xrightarrow{a(\vec{t})} \mu \qquad q \xrightarrow{b(\vec{t})} \mu'}{p \,||\, q \xrightarrow{c(\vec{t})} \mu''} \ \text{if } \gamma(a, b) = c, \ \forall p', q' \,.\, \mu''(p' \,||\, q') = \mu(p') \cdot \mu'(q')$$

Operational semantics of parallel composition

$$X(n : \{2,3\}) = \text{write}(n) \cdot X(n) + c \sum_{n' : \{2,3\}} \frac{1}{2} : X(n')$$

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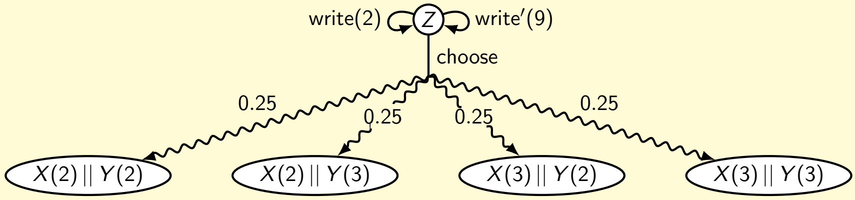

Linearisation of parallel composition

Linearisation of $X \,||\, Y$ is done compositionally: we first transform X and Y to LPPE, and then put them in parallel.

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$$\begin{aligned} Z(g, g') = &\sum_{d} c \Rightarrow a(b) \sum\!\!\!\!\bullet_{e} \; f : Z(n, g') \\ &+ \sum_{d'} c' \Rightarrow a'(b') \sum\!\!\!\!\bullet_{e'} \; f' : Z(g, n') \\ &+ \sum_{(d,d')} c \wedge c' \wedge b = b' \Rightarrow \gamma(a, a')(b) \sum\!\!\!\!\bullet_{(e,e')} \; f \cdot f' : Z(n, n') \end{aligned}$$

Linearisation of hiding, encapsulation and renaming

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&+ \sum_{d_2 : D_2} c_2 \Rightarrow a_2(b_2) \sum\!\!\!\!\bullet_{e_2 : E_2} f_2 : X(n_2) \\
&+ \sum_{d_3 : D_3} c_3 \Rightarrow a_3(b_3) \sum\!\!\!\!\bullet_{e_3 : E_3} f_3 : X(n_3)
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Linearisation of hiding, encapsulation and renaming is also done compositionally.

$$\begin{aligned}\tau_{\{a_2\}}(X(g : G)) = &\sum_{d_1:D_1} c_1 \Rightarrow a_1(b_1) \sum\!\!\!\bullet_{e_1:E_1} f_1 : X(n_1) \\ &+ \sum_{d_2:D_2} c_2 \Rightarrow a_2(b_2) \sum\!\!\!\bullet_{e_2:E_2} f_2 : X(n_2) \\ &+ \sum_{d_3:D_3} c_3 \Rightarrow a_3(b_3) \sum\!\!\!\bullet_{e_3:E_3} f_3 : X(n_3)\end{aligned}$$

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Case study: a leader election protocol

Implementation

- Haskell: one-to-one mapping of algorithms to code
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- Two processes throw a coin
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Case study: leader election protocol à la Itai-Rodeh

- Two processes throw a coin
  - *Both heads or both tails → throw again*
  - *One of them heads → this will be the leader*
- More precise:
  - *Passive thread: receive value of opponent*
  - *Active thread: flip, send, compare (or block)*

A prCRL model of the leader election protocol

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&\qquad (\ (d = e \Rightarrow A(id)) \\
&\qquad +(d = heads \wedge d \neq e \Rightarrow leader(id) \cdot A(id)) \\
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&\quad flip(id) \sum_{d:D} \bullet \tfrac{1}{2} : send(other(id), id, d) \cdot \sum_{e:D} readVal(e). \\
&\qquad (\ (d = e \Rightarrow A(id)) \\
&\qquad +(d = heads \wedge d \neq e \Rightarrow leader(id) \cdot A(id)) \\
&\qquad +(d = tails \wedge d \neq e \Rightarrow follower(id) \cdot A(id))) \\
&C(id:Id) = \qquad\qquad P(id, heads, false) \,||\, A(id)
\end{aligned}
$$

A prCRL model of the leader election protocol

$$P(id : \{1,2\}, val : D, set : Bool) =$$
$$set = false \Rightarrow \sum_{d:D} rec(id, other(id), d) \cdot P(id, d, true))$$
$$+ \; set = true \Rightarrow getVal(val).P(id, val, false)$$
$$A(id : Id) =$$
$$flip(id) \sum_{d:D}\!\!\bullet \; \tfrac{1}{2} : send(other(id), id, d) \cdot \sum_{e:D} readVal(e).$$
$$(\; (d = e \Rightarrow A(id))$$
$$+(d = heads \wedge d \neq e \Rightarrow leader(id) \cdot A(id))$$
$$+(d = tails \wedge d \neq e \Rightarrow follower(id) \cdot A(id)))$$
$$C(id : Id) = \qquad P(id, heads, false) \,||\, A(id)$$

$$\gamma(getVal, readVal) = checkVal$$

A prCRL model of the leader election protocol

$$
\begin{aligned}
&P(id : \{1,2\}, val : D, set : Bool) = \\
&\qquad set = false \Rightarrow \sum_{d:D} rec(id, other(id), d) \cdot P(id, d, true)) \\
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&A(id : Id) = \\
&\quad flip(id) \sum_{d:D}\!\!\bullet \tfrac{1}{2} : send(other(id), id, d) \cdot \sum_{e:D} readVal(e). \\
&\qquad (\ (d = e \Rightarrow A(id)) \\
&\qquad +(d = heads \wedge d \neq e \Rightarrow leader(id) \cdot A(id)) \\
&\qquad +(d = tails \wedge d \neq e \Rightarrow follower(id) \cdot A(id))) \\
&C(id : Id) = \partial_{getVal, readVal}(P(id, heads, false) \,||\, A(id))
\end{aligned}
$$

$$\gamma(getVal, readVal) = checkVal$$

A prCRL model of the leader election protocol

$$
\begin{aligned}
&P(id : \{1,2\}, val : D, set : Bool) = \\
&\qquad set = false \Rightarrow \sum_{d:D} rec(id, other(id), d) \cdot P(id, d, true)) \\
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&\qquad +(d = heads \wedge d \neq e \Rightarrow leader(id) \cdot A(id)) \\
&\qquad +(d = tails \wedge d \neq e \Rightarrow follower(id) \cdot A(id))) \\
&C(id : Id) = \partial_{getVal, readVal}(P(id, heads, false) \,||\, A(id)) \\
&S = \qquad C(1) \,||\, C(2)
\end{aligned}
$$

$$\gamma(getVal, readVal) = checkVal$$

A prCRL model of the leader election protocol

$$
\begin{aligned}
&P(id : \{1,2\}, val : D, set : Bool) = \\
&\qquad set = false \Rightarrow \sum_{d:D} rec(id, other(id), d) \cdot P(id, d, true)) \\
&\quad + set = true \Rightarrow getVal(val).P(id, val, false) \\
&A(id : Id) = \\
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&\qquad (\ (d = e \Rightarrow A(id)) \\
&\qquad +(d = heads \wedge d \neq e \Rightarrow leader(id) \cdot A(id)) \\
&\qquad +(d = tails \wedge d \neq e \Rightarrow follower(id) \cdot A(id))) \\
&C(id : Id) = \partial_{getVal, readVal}(P(id, heads, false) \,||\, A(id)) \\
&S = \qquad\qquad C(1) \,||\, C(2)
\end{aligned}
$$

$$\gamma(rec, send) = comm \qquad \gamma(getVal, readVal) = checkVal$$

A prCRL model of the leader election protocol

$$
\begin{aligned}
& P(id : \{1,2\}, val : D, set : Bool) = \\
& \qquad set = false \Rightarrow \sum_{d:D} rec(id, other(id), d) \cdot P(id, d, true)) \\
& \quad + set = true \Rightarrow getVal(val).P(id, val, false) \\
& A(id : Id) = \\
& \quad flip(id) \sum_{d:D} \bullet \tfrac{1}{2} : send(other(id), id, d) \cdot \sum_{e:D} readVal(e). \\
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& \qquad +(d = heads \wedge d \neq e \Rightarrow leader(id) \cdot A(id)) \\
& \qquad +(d = tails \wedge d \neq e \Rightarrow follower(id) \cdot A(id))) \\
& C(id : Id) = \partial_{getVal, readVal}(P(id, heads, false) \,||\, A(id)) \\
& S = \partial_{send, rec}(C(1) \,||\, C(2))
\end{aligned}
$$

$$\gamma(rec, send) = comm \qquad \gamma(getVal, readVal) = checkVal$$

Reductions on the leader election protocol model

In order to obtain reductions: first linearise

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
$\rightarrow$ LPPE with 18 parameters and 14 summands

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
$\rightarrow$ LPPE with 18 parameters and 14 summands

$$Z(pc11 : \{1..1\}, id11 : Ids, val11 : D, set11 : Bool, d11 : D, pc21 : \{1..4\}, id21 : Id, d21 : D, e21 : D, pc12 : \{1..1\}, id12 : Ids, val12 : D, set12 : Bool, d12 : D, pc22 : \{1..4\}, id22 : Id, d22 : D, e22 : D) =$$

$$\ldots$$

$$\sum_{e21:D} pc21 = 3 \land pc11 = 1 \land set11 \land val11 = e21 \Rightarrow checkVal(val11) \sum\!\!\!\bullet_{(k1,k2):\{*\}\times\{*\}} multiply(1.0, 1.0) : Z(1, id11, val11, false, tails, 4, id21, d21, e21, pc12, id12, val12, set12, d12, pc22, id22, d22, e22)$$

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
$\rightarrow$ LPPE with 18 parameters and 14 summands

$$
\begin{aligned}
&Z(pc11 : \{1..1\}, id11 : Ids, val11 : D, set11 : Bool, d11 : D, pc21 : \{1..4\},\\
&\quad id21 : Id, d21 : D, e21 : D, pc12 : \{1..1\}, id12 : Ids, val12 : D,\\
&\quad set12 : Bool, d12 : D, pc22 : \{1..4\}, id22 : Id, d22 : D, e22 : D) =\\
&\ldots\\
&\sum_{e21:D} pc21 = 3 \wedge pc11 = 1 \wedge set11 \wedge val11 = e21 \Rightarrow\\
&\quad checkVal(val11) \sum_{(k1,k2):\{*\}\times\{*\}} multiply(1.0, 1.0) :\\
&\quad Z(1, id11, val11, false, tails, 4, id21, d21, e21,\\
&\qquad pc12, id12, val12, set12, d12, pc22, id22, d22, e22)
\end{aligned}
$$

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
$\rightarrow$ LPPE with 18 parameters and 14 summands

$$Z(pc11 : \{1..1\}, id11 : Ids, val11 : D, set11 : Bool, d11 : D, pc21 : \{1..4\}, id21 : Id, d21 : D, e21 : D, pc12 : \{1..1\}, id12 : Ids, val12 : D, set12 : Bool, d12 : D, pc22 : \{1..4\}, id22 : Id, d22 : D, e22 : D) =$$

$$\ldots \sum_{e21:D} pc21 = 3 \wedge pc11 = 1 \wedge set11 \wedge val11 = e21 \Rightarrow checkVal(val11) \sum\!\!\!\!\bullet_{(k1,k2):\{*\}\times\{*\}} multiply(1.0, 1.0) : Z(1, id11, val11, false, tails, 4, id21, d21, e21, pc12, id12, val12, set12, d12, pc22, id22, d22, e22)$$

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
$\rightarrow$ LPPE with 18 parameters and 14 summands

$$Z(pc11 : \{1..1\}, id11 : Ids, val11 : D, set11 : Bool, d11 : D, pc21 : \{1..4\}, id21 : Id, d21 : D, e21 : D, pc12 : \{1..1\}, id12 : Ids, val12 : D, set12 : Bool, d12 : D, pc22 : \{1..4\}, id22 : Id, d22 : D, e22 : D) =$$

$$\ldots$$

$$\sum_{e21:D} pc21 = 3 \land pc11 = 1 \land set11 \land val11 = e21 \Rightarrow checkVal(val11) \sum_{(k1,k2):\{*\}\times\{*\}}^{\bullet} multiply(1.0, 1.0) : Z(1, id11, val11, false, tails, 4, id21, d21, e21, pc12, id12, val12, set12, d12, pc22, id22, d22, e22)$$

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
→ LPPE with 18 parameters and 14 summands

$$Z(pc11 : \{1..1\}, id11 : Ids, val11 : D, set11 : Bool, d11 : D, pc21 : \{1..4\},$$
$$id21 : Id, d21 : D, e21 : D, pc12 : \{1..1\}, id12 : Ids, val12 : D,$$
$$set12 : Bool, d12 : D, pc22 : \{1..4\}, id22 : Id, d22 : D, e22 : D) =$$

$$\ldots$$

$$\sum_{e21:D} pc21 = 3 \wedge pc11 = 1 \wedge set11 \wedge val11 = e21 \Rightarrow$$
$$checkVal(val11) \sum_{(k1,k2):\{*\}\times\{*\}}^{\bullet} multiply(1.0, 1.0) :$$
$$Z(1, id11, val11, false, tails, 4, id21, d21, e21,$$
$$pc12, id12, val12, set12, d12, pc22, id22, d22, e22)$$

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
$\rightarrow$ LPPE with 18 parameters and 14 summands

$$Z(pc11 : \{1..1\}, id11 : Ids, val11 : D, set11 : Bool, d11 : D, pc21 : \{1..4\},$$
$$id21 : Id, d21 : D, e21 : D, pc12 : \{1..1\}, id12 : Ids, val12 : D,$$
$$set12 : Bool, d12 : D, pc22 : \{1..4\}, id22 : Id, d22 : D, e22 : D) =$$

$\ldots$

$$\sum_{e21:D} pc21 = 3 \wedge pc11 = 1 \wedge set11 \wedge val11 = e21 \Rightarrow$$
$$checkVal(val11) \sum\!\!\!\!\bullet_{(k1,k2):\{*\}\times\{*\}} multiply(1.0, 1.0) :$$
$$Z(1, id11, val11, false, tails, 4, id21, d21, e21,$$
$$pc12, id12, val12, set12, d12, pc22, id22, d22, e22)$$

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
$\rightarrow$ LPPE with 10 parameters and 12 summands

$$
\begin{aligned}
&Z(\mathit{val11} : D, \mathit{set11} : \mathit{Bool}, \mathit{pc21} : 1..4, \mathit{d21} : D, \mathit{e21} : D,\\
&\quad \mathit{val12} : D, \mathit{set12} : \mathit{Bool}, \mathit{pc22} : 1..4, \mathit{d22} : D, \mathit{e22} : D) =\\
&\quad \ldots\\
&\quad \mathit{pc21} = 3 \wedge \mathit{set11} \Rightarrow \mathit{checkVal}(\mathit{val11}) \sum_{k:\{*\}}\!\bullet\, 1.0 :\\
&\quad Z(\mathit{heads}, \mathit{false}, 4, \mathit{d21}, \mathit{val11}, \mathit{val12}, \mathit{set12}, \mathit{pc22}, \mathit{d22}, \mathit{e22})
\end{aligned}
$$

Reductions on the leader election protocol model

In order to obtain reductions: first linearise
$\rightarrow$ LPPE with 10 parameters and 12 summands

$$
\begin{aligned}
&Z(\mathit{val11} : D, \mathit{set11} : \mathit{Bool}, \mathit{pc21} : 1..4, \mathit{d21} : D, \mathit{e21} : D,\\
&\quad \mathit{val12} : D, \mathit{set12} : \mathit{Bool}, \mathit{pc22} : 1..4, \mathit{d22} : D, \mathit{e22} : D) =\\
&\quad \ldots\\
&\quad \mathit{pc21} = 3 \wedge \mathit{set11} \Rightarrow \mathit{checkVal}(\mathit{val11}) \sum_{k:\{*\}}\!\!\bullet\; 1.0 :\\
&\quad Z(\mathit{heads}, \mathit{false}, 4, \mathit{d21}, \mathit{val11}, \mathit{val12}, \mathit{set12}, \mathit{pc22}, \mathit{d22}, \mathit{e22})
\end{aligned}
$$

State space: from 127 to 93 states.

Conclusions and Future Work

Conclusions / Results

- We developed the process algebra prCRL, incorporating both data and probability.
- We defined a linear format for prCRL, the LPPE, providing the starting point for effective symbolic optimisations and easy state space generation.
- We provided a linearisation algorithm to transform prCRL specifications to LPPEs, proved it correct, and implemented it.

Future work

Applying existing optimisation techniques to LPPEs

- Automating the translation from LPPE to PA
- Branching bisimulation preserving reductions (e.g., confluence reduction)

Questions

Questions?