UNIVERSITY OF TWENTE.

Formal Methods & Tools.

State Space Reduction of Linear Processes using Control Flow Reconstruction

Mark Timmer

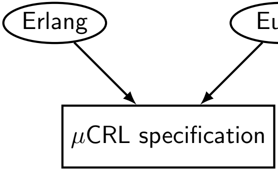
October 14, 2009

ATVA 2009

Joint work with Jaco van de Pol

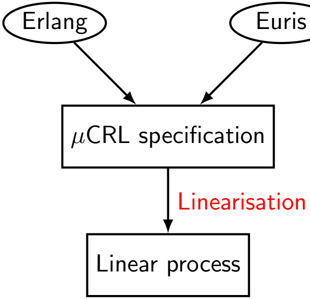
Contents

The μCRL toolset



System specification consisting of parallel processes

The μCRL toolset



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Intermediate format

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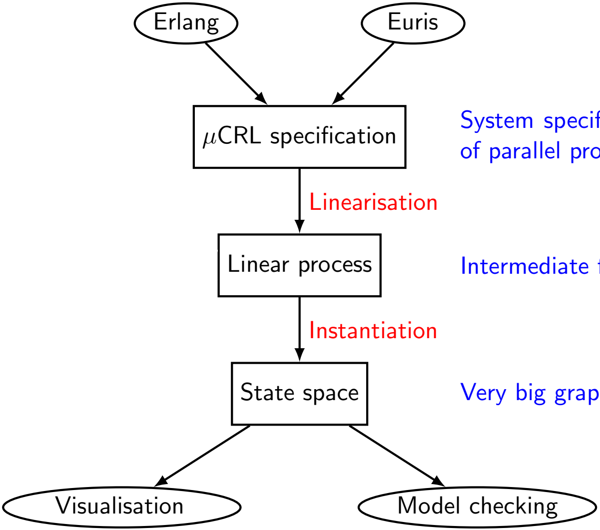

System specification consisting of parallel processes

Intermediate format

Very big graph

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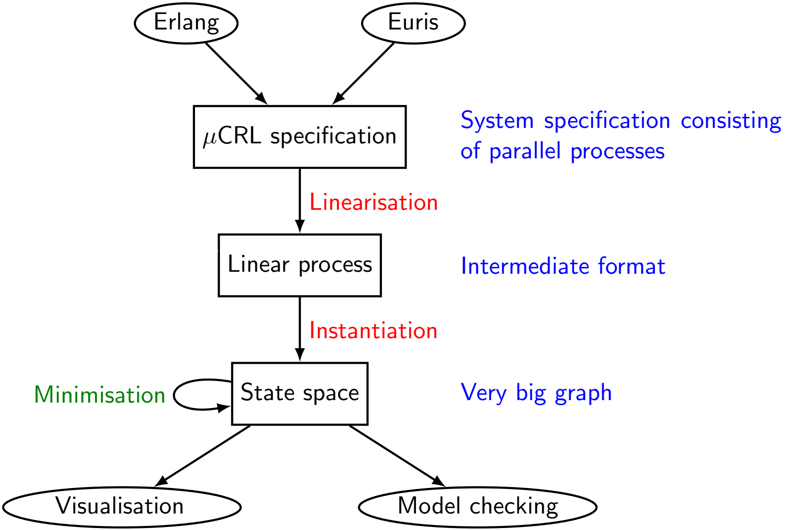

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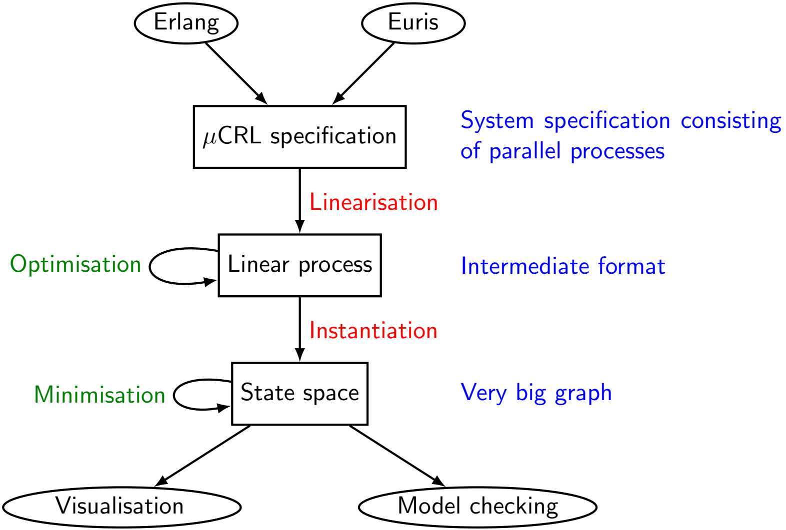

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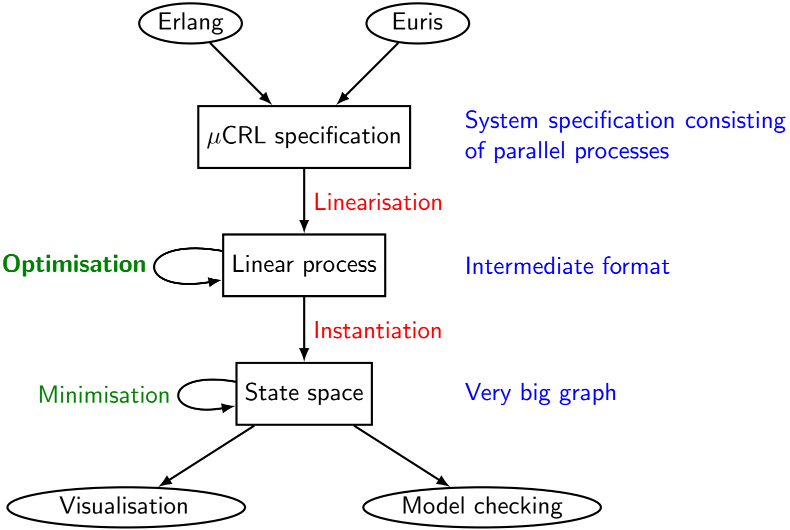

The linear process equation

The basic structure of an LPE

$$\begin{aligned} X(\boldsymbol{d} : \boldsymbol{D}) = & \sum_{\boldsymbol{e_1} : \boldsymbol{E_1}} c_1(\boldsymbol{d}, \boldsymbol{e_1}) \Rightarrow a_1(\boldsymbol{d}, \boldsymbol{e_1}) \cdot X(g_1(\boldsymbol{d}, \boldsymbol{e_1})) \\ + & \ \dots \\ + & \sum_{\boldsymbol{e_n} : \boldsymbol{E_n}} c_n(\boldsymbol{d}, \boldsymbol{e_n}) \Rightarrow a_n(\boldsymbol{d}, \boldsymbol{e_n}) \cdot X(g_n(\boldsymbol{d}, \boldsymbol{e_n})) \end{aligned}$$

- d: a vector of global state variables
- e_i: a vector of local variables for summand i
- c_i: the enabling condition for summand i
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\boldsymbol{d} \xrightarrow{a(\boldsymbol{p})} \boldsymbol{d}' \Leftrightarrow \exists i \,.\, \exists \boldsymbol{e_i} \,.\, c_i(\boldsymbol{d}, \boldsymbol{e_i}) = \texttt{true} \land a_i(\boldsymbol{d}, \boldsymbol{e_i}) = a(\boldsymbol{p}) \land g_i(\boldsymbol{d}, \boldsymbol{e_i}) = \boldsymbol{d}'
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An example

$$X = \sum_{m:\{m_1,\ldots,m_{10}\}} \mathsf{in}(m) \cdot (\tau \cdot \mathsf{loss} \cdot X + \tau \cdot \mathsf{out}(m) \cdot X)$$

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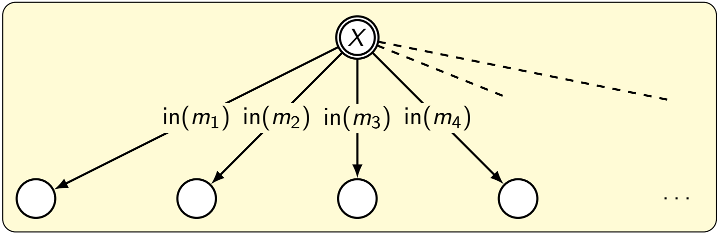

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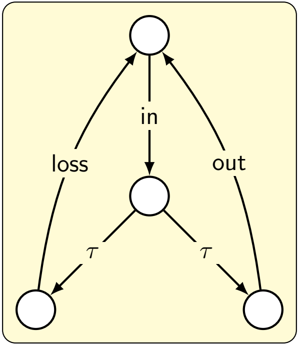

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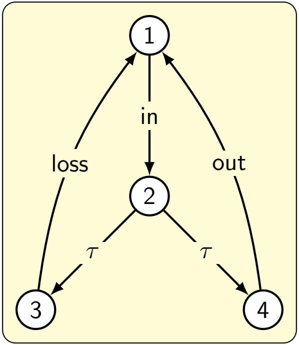

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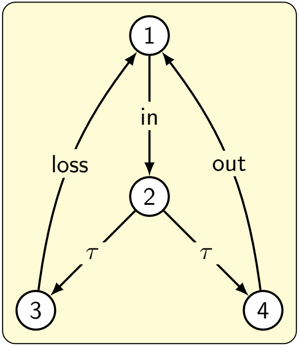

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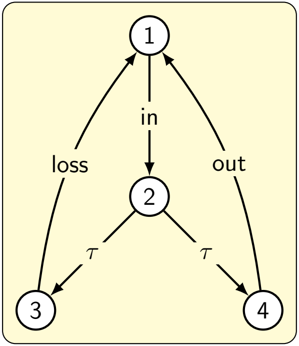

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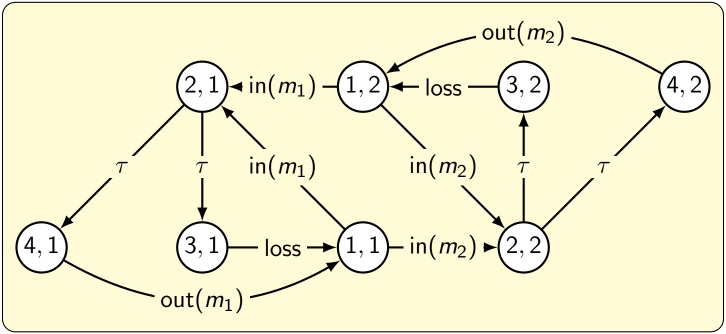
Initial process: $X(1, m_1)$.

An example


out(m_2)
2, 1
in(m_1)
1, 2
loss
3, 2
4, 2
τ
τ
in(m_1)
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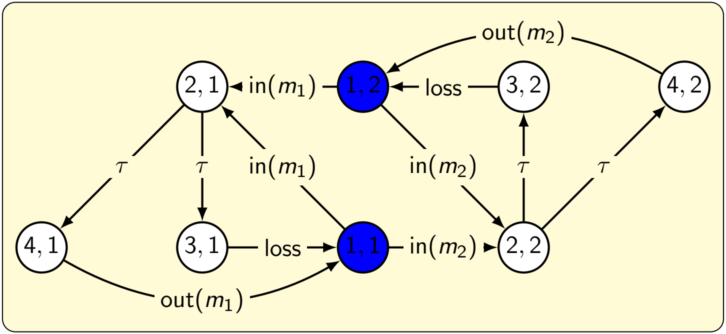
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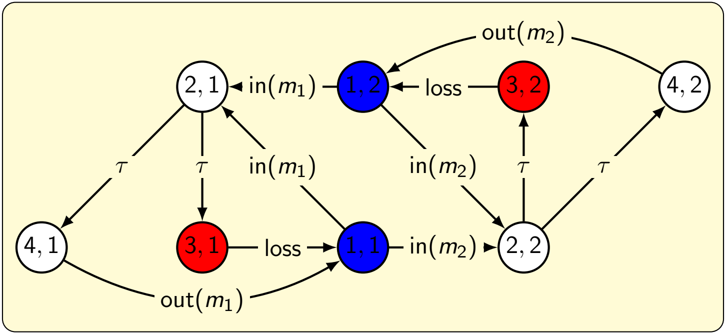
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τ
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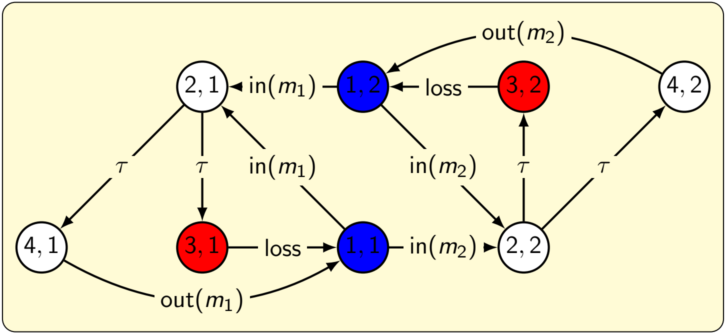
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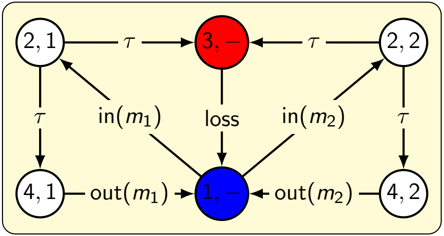
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τ
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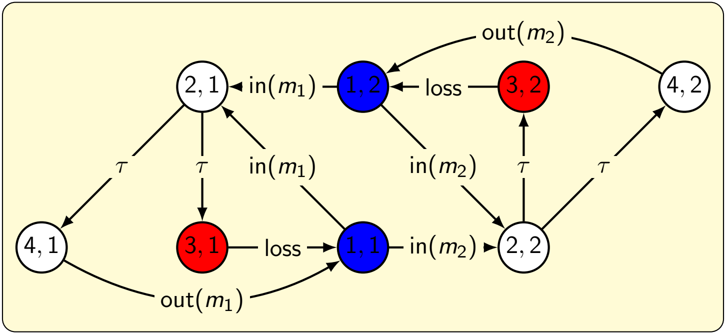

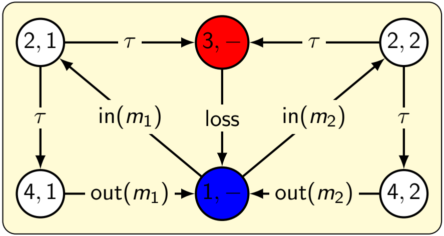

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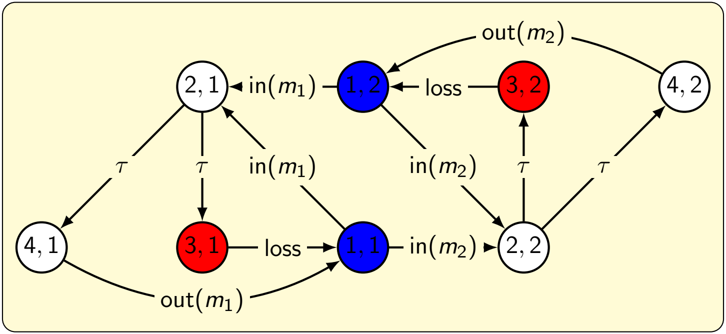

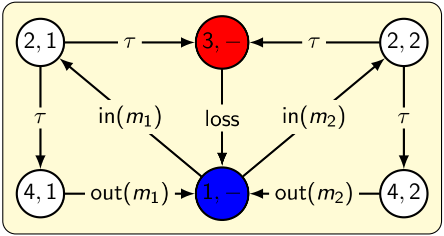

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Control Flow Reconstruction

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Problem: control flow is hidden in state parameters. Moreover, there are several control flows due to parallelism.

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Solution:

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Problem: control flow is hidden in state parameters. Moreover, there are several control flows due to parallelism.

Solution:

1. Detect control flow parameters
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4. Deduce when data parameters are (globally) relevant
5. Transform the LPE

Control flow parameters

Observation: program counters (control flow parameters) are special.

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$B_1 = \sum_{d\colon D} \mathit{read}(d) \cdot w(d) \cdot B_1$ $B_2 = \sum_{d\colon D} r(d) \cdot \mathit{write}(d) \cdot B_2$

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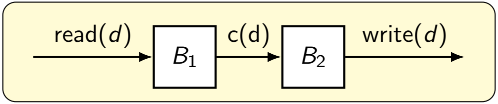

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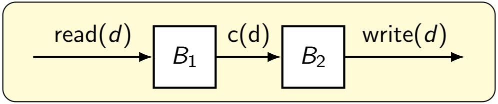

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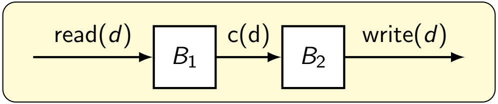

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In every summand, each control flow parameter

- is either left unchanged, or
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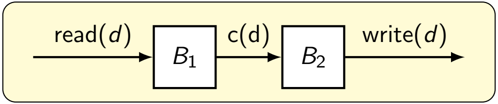

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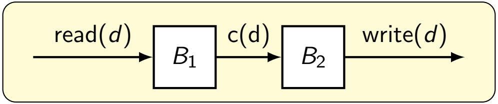

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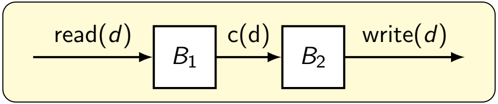

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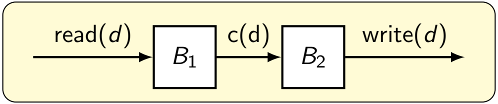

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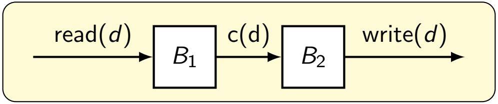
Control Flow Graph



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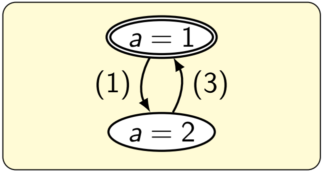

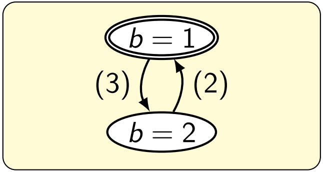

The *belongs to* relation

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A data parameter k belongs to a CFP j if the cluster of j contains all summands that

- either change k, or
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So, x belongs to a and y belongs to b. Thus, relevance of x can be decided by looking only at the control flow of a.

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$R(k, j, s)$: parameter k is relevant when CFP j is in state s

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There is a summand that can be taken when $d_j = s$, that either

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$R(k, j, s)$: parameter k is relevant when CFP j is in state $s \iff$

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If $\neg R(k, j, s)$, then k is irrelevant when j is in state s

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For $|D| = 5$: state space reduction from 60 to 36 states.

Correctness and effectiveness

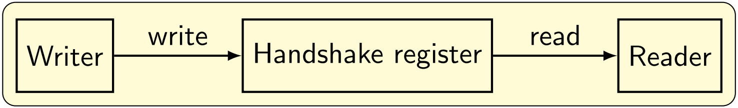
Theorem: correctness

The transformed LPE is strongly bisimilar to the original

Theorem: effectiveness

The number of reachable states of the transformed LPE is at most as large as the number of reachable states in the original

Case study: a handshake register



- Recentness

  Any value read was at some point during reading the last value written

- Sequentiality

  The values of sequential reads occur in the same order as they were written

- Waitfree

  Completion of reads/writes in a bounded number of steps

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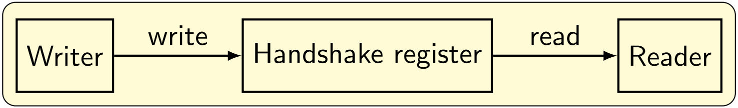

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Building blocks:

- 4x safe register (random read during writing)
- 4x atomic boolean register

Verifying the implementation

- Model the handshake register specification as a μCRL process
- Model the implementation as a μCRL process
- Generate their state spaces
- Minimise with respect to some equivalence
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Problem: state space explosion

Solution: Apply `stategraph`! (and compare to `parelm`)

Applying stategraph

	constelm | parelm | constelm		
	states	time (expl.)	time (symb.)
$\|D\| = 2$	540,736	0:23.0	0:04.5
$\|D\| = 3$	13,834,800	10:10.3	0:06.7
$\|D\| = 4$	142,081,536	–	0:09.0
$\|D\| = 5$	883,738,000	–	0:11.9
$\|D\| = 6$	3,991,840,704	–	0:15.4

	constelm | stategraph | constelm		
	states	time (expl.)	time (symb.)
$\|D\| = 2$	45,504	0:02.4	0:01.3
$\|D\| = 3$	290,736	0:12.7	0:01.4
$\|D\| = 4$	1,107,456	0:48.9	0:01.6
$\|D\| = 5$	3,162,000	2:20.3	0:01.8
$\|D\| = 6$	7,504,704	5:26.1	0:01.9

Other case studies

Other specifications `stategraph` was applied to:

- An Automatic In-flight Data Acquisition unit for a helicopter
- A cache coherence protocol for a distributed JVM
- The sliding window protocol
- An automatic translation from Erlang to μCRL of a distributed resource locker in Ericsson's AXD 301 switch

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Results:

- Reductions in the number of states (up to 20 percent)
- Reductions in the number of parameters (up to 75 percent)
- Reductions in the number of summands (up to 25 percent)

Conclusions and Future Work

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- Novel method for reconstructing control flow
  - Even control flow hiding in state parameters is found
- Data flow analysis based on this control flow
  - Resetting variables that are no longer relevant (globally!)
  - Decreases in states, parameters and summands
  - Reductions obtained before generating the entire state space
- Precise proofs of correctness *and* decrease of state space
- Case studies show that impressive results are indeed obtained

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Future work:

- Investigate additional applications for the reconstructed control flow
  - Invariant generation
  - Visualisation (already implemented)
  - Improve confluence checking
- Use more precise abstractions based on control flow
- Apply these techniques to a probabilistic linear format

Questions

?