UNIVERSITY OF TWENTE.

Formal Methods & Tools.

A linear process-algebraic format for probabilistic systems with data

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Joint work with Joost-Pieter Katoen, Jaco van de Pol, and Mariëlle Stoelinga

Probabilistic Model Checking

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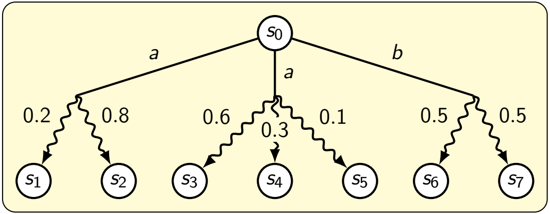

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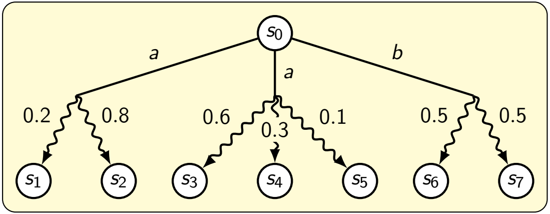

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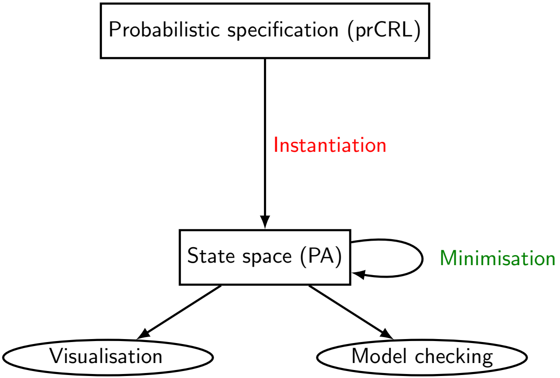
Limitations of previous approaches:

- Susceptible to the state space explosion problem
- Restricted treatment of data

Overview of our approach



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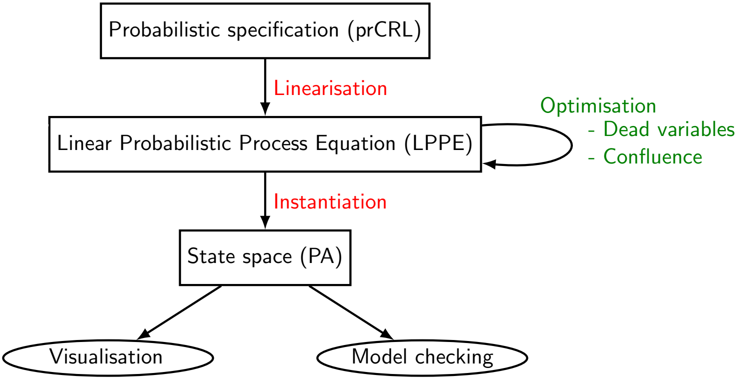

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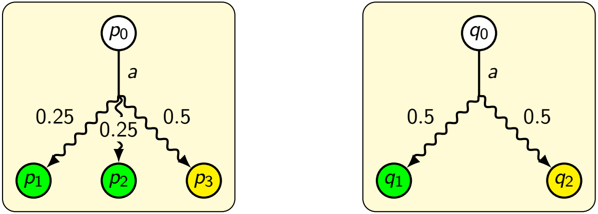

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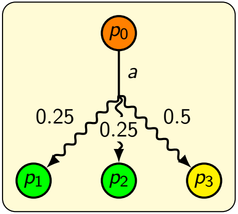

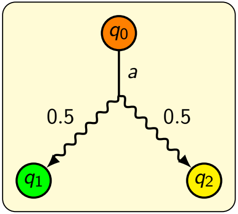

Contents

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A process algebra with data and probability: prCRL

Specification language prCRL:

- Based on μCRL (so data), with additional probabilistic choice
- Semantics defined in terms of probabilistic automata
- Minimal set of operators to facilitate formal manipulation
- Syntactic sugar easily definable

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The grammar of prCRL process terms

Process terms in prCRL are obtained by the following grammar:

$$p ::= Y(\vec{t}) \mid c \Rightarrow p \mid p + p \mid \sum_{x:D} p \mid a(\vec{t}) \sum_{x:D}\!\!\bullet\; f : p$$

Process equations and processes

A process equation is something of the form $X(\vec{g} : \vec{G}) = p$.

An example specification

Sending an arbitrary natural number

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\begin{aligned}
X(\text{active} : \text{Bool}) = \\
& \text{not}(\text{active}) \Rightarrow \text{ping} \cdot \sum_{b:\text{Bool}} X(b) \\
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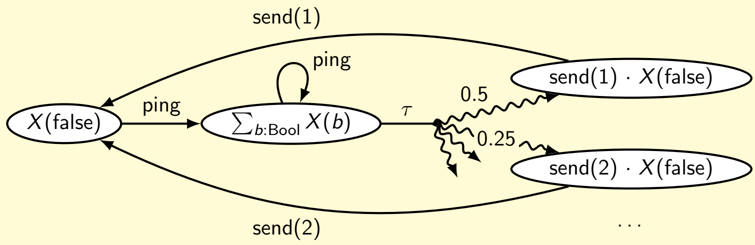

Compositionality using extended prCRL

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$$Y(m : \{1,2\}) = \text{write}_Y(m) \cdot Y(m) + \text{choose}' \sum_{m' : \{1,2\}}^{\bullet} \frac{1}{2} : Y(m')$$

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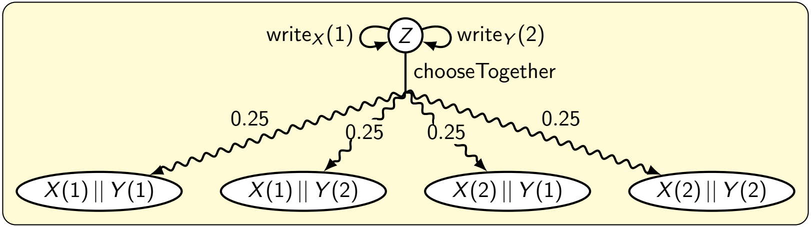

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A linear format for prCRL: the LPPE

LPPEs are a subset of prCRL specifications:

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Advantages of using LPPEs instead of prCRL specifications:

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- Straight-forward parallel composition
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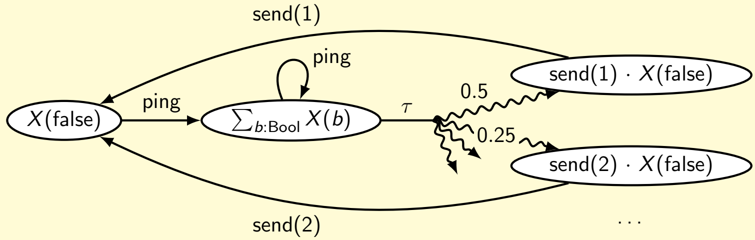
Theorem

Every specification (without unguarded recursion) can be linearised to an LPPE, preserving strong probabilistic bisimulation.

Linear Probabilistic Process Equations – an example



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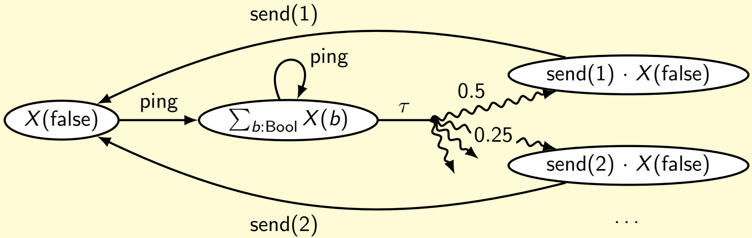

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Specification in LPPE

$$X(pc : \{1..3\}, n : \mathbb{N}^{\geq 0}) =$$

$$+ \; pc = 1 \Rightarrow \text{ping} \cdot X(2, 1)$$

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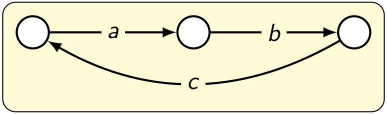

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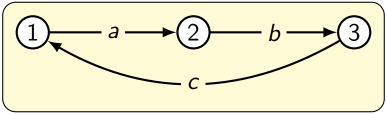

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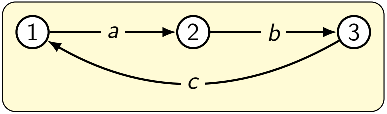

The corresponding LPPE (initialised with pc = 1):

$$\begin{aligned} Y(pc\colon \{1,2,3\}) = & \\ & pc = 1 \Rightarrow a \cdot Y(2) \\ + & \; pc = 2 \Rightarrow b \cdot Y(3) \\ + & \; pc = 3 \Rightarrow c \cdot Y(1) \end{aligned}$$

Linearisation: a more complicated example with data

Consider the following prCRL specification:

$$X = \sum_{d:D} \text{get}(d) \cdot (\tau \cdot \text{loss} \cdot X + \tau \cdot \text{send}(d) \cdot X)$$

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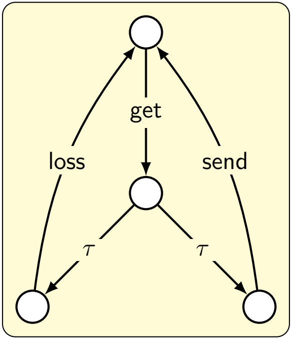

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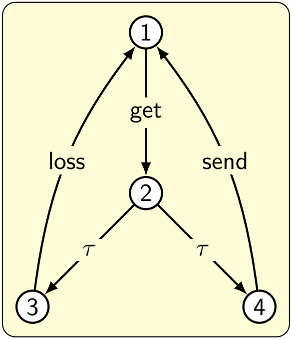

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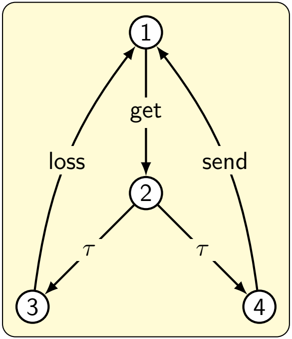

LPPE:

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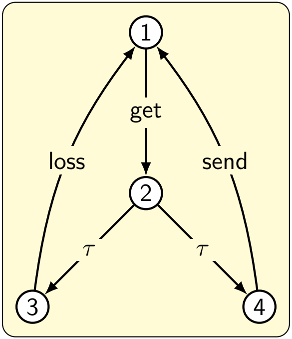

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Initial process: $Y(1, d_1)$.

Linearisation: a more algorithmic approach

Consider the following prCRL specification:

$$X(d:D) = \sum_{e:D} a(d+e) \sum\!\!\!\!\bullet_{f:D} \frac{1}{|D|} : \left(c(e) \cdot c(f) \cdot X(5) + c(e+f) \cdot X(5) \right)$$

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$$\begin{aligned} X(\mathrm{pc}:\{1,2,3\}, d:D, e:D, f:D) = \\ \mathrm{pc}=1 \Rightarrow \sum_{e:D} a(d+e) \sum_{f:D} \tfrac{1}{|D|} \cdot X(2,d,e,f) \\ + \mathrm{pc}=2 \Rightarrow c(e) \cdot X(3,d,e,f) \\ + \mathrm{pc}=2 \Rightarrow c(e+f) \cdot X(1,5,e,f) \\ + \mathrm{pc}=3 \Rightarrow c(f) \cdot X(1,5,e,f) \end{aligned}$$

Linearisation

In general, we always linearise in two steps:

1. Transform the specification to intermediate regular form (IRF) (every process is a summation of single-action terms)
2. Merge all processes into one big process by introducing a program counter

In the first step, global parameters are introduced to remember the values of bound variables.

Contents

Case study: a leader election protocol

- Implementation in Haskell:
  - Linearisation: from prCRL to LPPE
  - Parallel composition of LPPEs, hiding, renaming, encapsulation
  - Generation of the state space of an LPPE
  - Automatic constant elimination and summand simplification
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Leader election protocol à la Itai-Rodeh

- Two processes throw a die
  - *The process with the highest number will be leader*
  - *In case of a tie: throw again*

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Leader election protocol à la Itai-Rodeh

- Two processes throw a die
  - *The process with the highest number will be leader*
  - *In case of a tie: throw again*
- More precisely:
  - *Passive thread: receive value of opponent*
  - *Active thread: roll, send, compare (or block)*

A prCRL model of the leader election protocol

$$
\begin{aligned}
& P(id : \{one, two\}, val : Die, set : Bool) = \\
& \qquad set = false \Rightarrow \sum_{d:Die} communicate(id, other(id), d) \cdot P(id, d, true) \\
& \quad + set = true \Rightarrow checkValue(val) \cdot P(id, val, false) \\
& A(id : \{one, two\}) = \\
& \quad roll(id) \sum_{d:Die} \bullet \tfrac{1}{6} : \overline{communicate}(other(id), id, d) \cdot \sum_{e:Die} \overline{checkValue}(e) \cdot \\
& \qquad \big(\ (d = e \Rightarrow A(id)) \\
& \qquad + (d > e \Rightarrow leader(id) \cdot A(id)) \\
& \qquad + (e > d \Rightarrow follower(id) \cdot A(id))\big) \\
& C(id : \{one, two\}) = P(id, 1, false) \,||\, A(id) \\
& S = C(one) \,||\, C(two)
\end{aligned}
$$

Reductions on the leader election protocol model

In order to obtain reductions first linearise:

$$\sum_{e21:Die} pc21 = 3 \wedge pc11 = 1 \wedge set11 \wedge val11 = e21 \Rightarrow$$
$$checkValue(val11) \sum_{\bullet (k1,k2):\{*\}\times\{*\}} multiply(1.0, 1.0):$$
$$Z(1, id11, val11, false, 1, 4, id21, d21, e21,$$
$$pc12, id12, val12, set12, d12, pc22, id22, d22, e22)$$

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Before reductions:

- 18 parameters
- 14 summands
- 3763 states
- 6158 transitions

Reductions on the leader election protocol model

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&\quad \mathit{val12}, \mathit{set12}, \mathit{pc22}, \mathit{d22}, \mathit{e22})
\end{aligned}
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Before reductions:

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After reductions:

- 10 parameters
- 12 summands

Reductions on the leader election protocol model

In order to obtain reductions first linearise:

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$$checkValue(val11) \sum_{(k1,k2):\{*\}\times\{*\}} 1.0 :$$
$$Z(\quad 1 \quad, false, \quad 4, \quad d21, val11,$$
$$val12, set12, \quad pc22, \quad d22, e22)$$

Before reductions:

- 18 parameters
- 14 summands
- 3763 states
- 6158 transitions

After reductions:

- 10 parameters
- 12 summands
- 1693 states (-55%)
- 2438 transitions (-60%)

Contents

Conclusions and Future Work

Conclusions / Results

- We developed the process algebra prCRL, incorporating both data and probability.
- We defined a normal form for prCRL, the LPPE; starting point for symbolic optimisations and easy state space generation.
- We provided a linearisation algorithm to transform prCRL specifications to LPPEs, proved it correct, implemented it, and used it to show significant reductions on a case study.

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Future work

- Develop additional reduction techniques, for instance confluence reduction (in progress).
- Generalise proof techniques such as cones and foci to the probabilistic case.

Questions

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